(ii) Given : $2x^2+x-4=0$

Here, $a=2,\ b=1,\ c=-4$ [On comparing with $ax^2+bx+c=0$]

$\therefore$ $D=b^2-4ac=1^2-4\times2\times(-4)=1+32=33>0$

So, the given equation has real roots which are given by

$$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-1+\sqrt{33}}{2\times2}=\frac{-1+\sqrt{33}}{4};\ \beta=\frac{-b-\sqrt{D}}{2a}=\frac{-1-\sqrt{33}}{4}$$

Hence, the roots of the given equation are $\frac{-1+\sqrt{33}}{4}$ and $\frac{-1-\sqrt{33}}{4}$.

(iii) Given : $4x^2+4\sqrt{3}x+3=0$

Here, $a=4,\ b=4\sqrt{3},\ c=3$ [On comparing with $ax^2+bx+c=0$]

$\therefore$ $D=b^2-4ac=(4\sqrt{3})^2-4\times4\times3=48-48=0$

So, the given equation has real roots which are given by

$$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{-4\sqrt{3}+0}{2\times4}=-\frac{\sqrt{3}}{2};\ \beta=\frac{-b-\sqrt{D}}{2a}=\frac{-4\sqrt{3}-0}{2\times4}=-\frac{\sqrt{3}}{2}$$

Hence, the given quadratic equation has roots $-\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$.

(iv) Given : $2x^2+x+4=0$. Here, $a=2,\ b=1,\ c=4$

$\therefore$ $D=b^2-4ac=1^2-4\times2\times4=-31<0$

As discriminant D is negative, the given quadratic equation has **no real roots.**

mple 48 Find the roots of the quadratic equations by using the quadratic formula in eac he following :

(i) $2x^2-3x-5=0$ (ii) $5x^2+13x+8=0$ (iii) $-3x^2+5x+12=0$

(iv) $-x^2+7x-10=0$ (v) $x^2+2\sqrt{2}x-6=0$ (vi) $x^2-3\sqrt{5}x+10=0$

(vii) $\frac{1}{2}x^2-\sqrt{11}x+1=0$ [Exemplar Problem]

lution (i) Given : $2x^2-3x-5=0$. Here, $a=2,\ b=-3,\ c=-5$

Solved Examples

Example 12 Find the roots of the following quadratic equations (if they exist) by the method of completing the squares :

(i) $2x^2 - 7x + 3 = 0$ (ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$ (iv) $2x^2 + x + 4 = 0$ [NCERT]

Solution. (i) Given : $2x^2 - 7x + 3 = 0$

$\Rightarrow$ $2x^2 - 7x = -3$ [Transferring the constant term]

$\Rightarrow$ $x^2 - \frac{7}{2}x = -\frac{3}{2}$ [Dividing both sides by 2]

Adding $\left[\frac{1}{2}\text{ coefficient of } x\right]^2$ i.e., $\left[\frac{1}{2}\times\left(-\frac{7}{2}\right)\right]^2$ on both sides, we get

$$x^2 - \frac{7}{2}x + \left(-\frac{7}{4}\right)^2 = \left(-\frac{7}{4}\right)^2 - \frac{3}{2}$$

$$\left(x - \frac{7}{4}\right)^2 = \frac{49-24}{16} = \frac{25}{16} = \left(\frac{5}{4}\right)^2$$

$\Rightarrow$ $x - \frac{7}{4} = \pm\frac{5}{4}$ $\Rightarrow$ $x = \frac{7}{4} \pm \frac{5}{4}$

$\Rightarrow$ $x = \frac{7}{4} + \frac{5}{4}$ or $x = \frac{7}{4} - \frac{5}{4}$

$\Rightarrow$ $x = \frac{7+5}{4} = 3$ or $x = \frac{7-5}{4} = \frac{2}{4} = \frac{1}{2}$

Hence, the roots of the equation are 3 and $\frac{1}{2}$.

(ii) Given : $2x^2 + x - 4 = 0$

$\Rightarrow$ $2x^2 + x = 4$

$\Rightarrow$ $x^2 + \frac{1}{2}x = \frac{4}{2}$ [Dividing both sides by 2]

Adding $\left[\frac{1}{2}\text{ coefficient of } x\right]^2$ i.e., $\left(\frac{1}{2}\times\frac{1}{2}\right)^2$, on both sides, we get

$x^2 + \frac{1}{2}x + \frac{1}{16} = \frac{1}{16} + 2$ $\Rightarrow$ $\left(x + \frac{1}{4}\right)^2 = \left(\frac{\sqrt{33}}{4}\right)^2$

$x + \frac{1}{4} = \pm\frac{\sqrt{33}}{4}$ $\Rightarrow$ $x = \frac{-1 \pm \sqrt{33}}{4}$

Hence, the roots of the equation are $\frac{-1+\sqrt{33}}{4}$ and $\frac{-1-\sqrt{33}}{4}$.

(iii) Given : $4x^2 + 4\sqrt{3}x + 3 = 0$

$\Rightarrow$ $4x^2 + 4\sqrt{3}x = -3$

$\Rightarrow$ $x^2 + \frac{4\sqrt{3}}{4}x = -\frac{3}{4}$ [Dividing both sides by 4]

Adding $\left[\frac{1}{2}\text{ coefficient of } x\right]^2$ i.e., $\left(\frac{1}{2}\times\frac{4\sqrt{3}}{4}\right)^2$ or $\frac{3}{4}$ on both sides, we get

EX- 4.3

$$x^2+\sqrt{3}x+\frac{3}{4}=\frac{3}{4}-\frac{3}{4}$$

$$\Rightarrow \quad \left(x+\frac{\sqrt{3}}{2}\right)^2=0 \quad \Rightarrow \quad x+\frac{\sqrt{3}}{2}=0$$

Hence, $x=-\frac{\sqrt{3}}{2}$ **is the root repeated twice.**

(*iv*) Given : $2x^2+x+4=0 \Rightarrow 2x^2+x=-4$

$$x^2+\frac{1}{2}x=-\frac{4}{2} \qquad \text{[Dividing both sides by 2]}$$

Adding $\left[\frac{1}{2}\text{ coefficient of } x\right]^2$ i.e., $\left(\frac{1}{2}\times\frac{1}{2}\right)^2$ or $\frac{1}{16}$ to both sides, we get

$$x^2+\frac{1}{2}x+\frac{1}{16}=\frac{1}{16}-2 \quad \Rightarrow \quad \left(x+\frac{1}{4}\right)^2=\frac{1-32}{16}=-\frac{31}{16}<0$$

But $\left(x+\frac{1}{4}\right)^2$ cannot be negative for any real value of x.

$\Rightarrow$ No real value of x can satisfy the given equation.

Hence, the given equation has **no real roots.**

Example 43 Find the roots of the equation $5x^2-6x-2=0$ by the method of completing the square.

Solution. Multiplying the given equation throughout by 5, we get

$$25x^2-30x-10=0$$

$$\Rightarrow \quad (5x)^2-2\times5x\times3+3^2-3^2-10=0 \qquad \text{[Adding and subtracting } 3^2\text{]}$$

$$\Rightarrow \quad (5x-3)^2-9-10=0 \quad \Rightarrow \quad (5x-3)^2=19$$

$$\Rightarrow \quad 5x-3=\pm\sqrt{19} \quad \Rightarrow \quad 5x=3\pm\sqrt{19}$$

$$\Rightarrow \quad x=\frac{3\pm\sqrt{19}}{5}$$

Hence, the roots are $\frac{3+\sqrt{19}}{5}$ and $\frac{3-\sqrt{19}}{5}$.

Example 44 Solve the equation $x^2-(\sqrt{2}+1)x+\sqrt{2}$ by the method of completing the square.

Solution. Given : $x^2-(\sqrt{2}+1)x+\sqrt{2}=0$

$$\Rightarrow \quad x^2-(\sqrt{2}+1)x=-\sqrt{2}$$

$$\Rightarrow \quad x^2-2\left(\frac{\sqrt{2}+1}{2}\right)x+\left(\frac{\sqrt{2}+1}{2}\right)^2=\left(\frac{\sqrt{2}+1}{2}\right)^2-\sqrt{2} \quad \Rightarrow \quad \left(x-\frac{\sqrt{2}+1}{2}\right)^2=\frac{2+1+2\sqrt{2}}{4}-\sqrt{2}$$

$$\Rightarrow \quad \left(x-\frac{\sqrt{2}+1}{2}\right)^2=\frac{2+1+2\sqrt{2}-4\sqrt{2}}{4}=\frac{2+1-2\sqrt{2}}{4}$$

Adding $\left[\frac{1}{2}\text{ coefficient of } x\right]^2$ i.e., $\left(\frac{1}{2}\times\frac{4\sqrt{3}}{4}\right)^2$ or $\frac{3}{4}$ on both sides, we get

EX- 4.3

$$x^2+\sqrt{3}x+\frac{3}{4}=\frac{3}{4}-\frac{3}{4}$$

$$\Rightarrow \quad \left(x+\frac{\sqrt{3}}{2}\right)^2=0 \quad \Rightarrow \quad x+\frac{\sqrt{3}}{2}=0$$

Hence, $x=-\frac{\sqrt{3}}{2}$ **is the root repeated twice.**

(*iv*) Given : $2x^2+x+4=0 \Rightarrow 2x^2+x=-4$

$$x^2+\frac{1}{2}x=-\frac{4}{2}$$ [Dividing both sides by 2]

Adding $\left[\frac{1}{2}\text{ coefficient of } x\right]^2$ i.e., $\left(\frac{1}{2}\times\frac{1}{2}\right)^2$ or $\frac{1}{16}$ to both sides, we get

$$x^2+\frac{1}{2}x+\frac{1}{16}=\frac{1}{16}-2 \quad \Rightarrow \quad \left(x+\frac{1}{4}\right)^2=\frac{1-32}{16}=-\frac{31}{16}<0$$

But $\left(x+\frac{1}{4}\right)^2$ cannot be negative for any real value of x.

$\Rightarrow$ No real value of x can satisfy the given equation.

Hence, the given equation has **no real roots.**

Example 43 Find the roots of the equation $5x^2-6x-2=0$ by the method of completing the square.

Solution. Multiplying the given equation throughout by 5, we get

$$25x^2-30x-10=0$$

$$\Rightarrow \quad (5x)^2-2\times 5x\times 3+3^2-3^2-10=0$$ [Adding and subtracting 3^2]

$$\Rightarrow \quad (5x-3)^2-9-10=0 \quad \Rightarrow \quad (5x-3)^2=19$$

$$\Rightarrow \quad 5x-3=\pm\sqrt{19} \quad \Rightarrow \quad 5x=3\pm\sqrt{19}$$

$$\Rightarrow \quad x=\frac{3\pm\sqrt{19}}{5}$$

Hence, the roots are $\frac{3+\sqrt{19}}{5}$ and $\frac{3-\sqrt{19}}{5}$.

Example 44 Solve the equation $x^2-(\sqrt{2}+1)x+\sqrt{2}$ by the method of completing the square.

Solution. Given : $x^2-(\sqrt{2}+1)x+\sqrt{2}=0$

$$\Rightarrow \quad x^2-(\sqrt{2}+1)x=-\sqrt{2}$$

$$\Rightarrow \quad x^2-2\left(\frac{\sqrt{2}+1}{2}\right)x+\left(\frac{\sqrt{2}+1}{2}\right)^2=\left(\frac{\sqrt{2}+1}{2}\right)^2-\sqrt{2} \quad \Rightarrow \quad \left(x-\frac{\sqrt{2}+1}{2}\right)^2=\frac{2+1+2\sqrt{2}}{4}-\sqrt{2}$$

$$\Rightarrow \quad \left(x-\frac{\sqrt{2}+1}{2}\right)^2=\frac{2+1+2\sqrt{2}-4\sqrt{2}}{4}=\frac{2+1-2\sqrt{2}}{4}$$

Solution of a Quadratic Equation by Using Quadratic Formula

Consider the quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$

Dividing throughout by a, we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \quad \Rightarrow \quad x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\Rightarrow \quad x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a} \qquad \left[\text{Adding } \left(\frac{b}{2a}\right)^2 \text{ to both sides}\right]$$

$$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\Rightarrow \quad x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}, \text{ when } b^2 - 4ac \geq 0$$

$$\Rightarrow \quad x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This is known as **quadratic formula** or **Shreedharcharay's** formula for finding the roots of a quadratic equation.

Hence, if $b^2 - 4ac \geq 0$, then the roots of the quadratic equation $ax^2 + bx + c$ are given by

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Discriminant. *For the quadratic equation $ax^2 + bx + c = 0$, the expression $D = (b^2 - 4ac)$ is called a discriminant.* In terms of discriminant D, the two roots are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} \text{ and } \beta = \frac{-b - \sqrt{D}}{2a}$$

NOTE If the discrimant $D = b^2 - 4ac < 0$, then the quadratic equation $ax^2 + bx + c = 0$ has no real roots.

EX. 4.2

Solved Examples

Q NO 2

Example Find the roots of the following quadratic equations, if they exist, by applying the quadratic formula :

(*i*) $2x^2 - 7x + 3 = 0$ (*ii*) $2x^2 + x - 4 = 0$

(*iii*) $4x^2 + 4\sqrt{3}x + 3 = 0$ (*iv*) $2x^2 + x + 4 = 0$

Solution. (*i*) Given : $2x^2 - 7x + 3 = 0$

Here, $a = 2, b = -7, c = 3$ [On comparing with $ax^2 + bx + c = 0$]

$$\therefore \quad D = b^2 - 4ac = (-7)^2 - 4 \times 2 \times 3 = 49 - 24 = 25 > 0$$

So, the given equation has real roots which are given by

$$\alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-7) + \sqrt{25}}{2 \times 2} = \frac{7 + 5}{4} = \frac{12}{4} = 3$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{7 - 5}{4} = \frac{2}{4} = \frac{1}{2}$$

Hence, the roots of the given equation are 3 and $\frac{1}{2}$.