

9/6/20

EX-32

13. Solving (1) & (2), we get
 $x = 1$ and $y = -2$
 Hence, $X = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $Y = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$ and the
 values of x, y and z are $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$
 Subst. $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ in (3) $\Rightarrow \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$
 $\Rightarrow \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$
 Hence, $X = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ and $Y = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$
 Now, $X = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ and $Y = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$
 $\Rightarrow X + Y = \begin{bmatrix} 1+4 \\ -2-3 \\ 3+0 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \\ 3 \end{bmatrix}$
 Hence, $X + Y = \begin{bmatrix} 5 \\ -5 \\ 3 \end{bmatrix}$
 14. If $P(X) = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, then show that
 $P(X) \cdot P(Y) = P(X+Y)$

Soln. $P(X) = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$
 $P(Y) = \begin{bmatrix} \cos y & -\sin y \\ \sin y & \cos y \end{bmatrix}$
 $P(X+Y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) \\ \sin(x+y) & \cos(x+y) \end{bmatrix}$
 Now, $P(X) \cdot P(Y) = \begin{bmatrix} \cos x \cos y + \sin x \sin y & -\cos x \sin y - \sin x \cos y \\ \sin x \cos y - \cos x \sin y & \sin x \sin y + \cos x \cos y \end{bmatrix}$
 $= \begin{bmatrix} \cos(x+y) & -\sin(x+y) \\ \sin(x+y) & \cos(x+y) \end{bmatrix}$
 $= P(X+Y)$
 Hence, $P(X) \cdot P(Y) = P(X+Y)$

14. Show that (i) $\begin{bmatrix} 8 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 8 & -1 \end{bmatrix}$
 (ii) $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ -1 & 1 & 0 \\ 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

Matrix Inverse

Subst. (i) Let $A = \begin{bmatrix} 8 & -1 \\ 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$
 $AB = \begin{bmatrix} 8 \times 2 + (-1) \times 3 & 8 \times 1 + (-1) \times 4 \\ 6 \times 2 + 7 \times 3 & 6 \times 1 + 7 \times 4 \end{bmatrix} = \begin{bmatrix} 13 & 4 \\ 27 & 34 \end{bmatrix}$
 and $BA = \begin{bmatrix} 2 \times 8 + 1 \times 6 & 2 \times (-1) + 1 \times 7 \\ 3 \times 8 + 4 \times 6 & 3 \times (-1) + 4 \times 7 \end{bmatrix} = \begin{bmatrix} 22 & 5 \\ 30 & 25 \end{bmatrix}$
 Hence, $AB \neq BA$
 (ii) Let $A = \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 1 & 0 \\ 2 & 3 & 4 \end{bmatrix}$
 $AB = \begin{bmatrix} 1 \times (-1) + 1 \times (-1) + 3 \times 2 & 1 \times 1 + 1 \times 1 + 3 \times 0 & 1 \times 0 + 1 \times 0 + 3 \times 4 \\ 0 \times (-1) + 1 \times (-1) + 0 \times 2 & 0 \times 1 + 1 \times 1 + 0 \times 0 & 0 \times 0 + 1 \times 0 + 0 \times 4 \\ 1 \times (-1) + 1 \times (-1) + 0 \times 2 & 1 \times 1 + 1 \times 1 + 0 \times 0 & 1 \times 0 + 1 \times 0 + 0 \times 4 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 12 \\ 0 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$
 and $BA = \begin{bmatrix} (-1) \times 1 + 1 \times 0 + 0 \times 1 & (-1) \times 1 + 1 \times 1 + 0 \times 1 & 0 \times 1 + 1 \times 0 + 0 \times 1 \\ (-1) \times 0 + 1 \times 1 + 0 \times 1 & (-1) \times 1 + 1 \times 1 + 0 \times 1 & 0 \times 0 + 1 \times 0 + 0 \times 1 \\ 2 \times 1 + 3 \times 1 + 4 \times 0 & 2 \times (-1) + 3 \times 1 + 4 \times 0 & 2 \times 0 + 3 \times 0 + 4 \times 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 5 & 1 & 4 \end{bmatrix}$
 Hence, $AB \neq BA$

15. Find A^{-1} if $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix}$
 Soln. We are given that $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix}$
 $A^{-1} = \frac{1}{|A|} \text{adj } A$
 $|A| = \begin{vmatrix} 2 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 1 & 0 \end{vmatrix} = 2(0-1) - 1(0-2) = -2 + 2 = 0$
 $\therefore |A| = 0$
 Hence, A^{-1} does not exist.

16. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
 find A^{-1} and B^{-1}
 Soln. We are given that $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
 $A^{-1} = \frac{1}{|A|} \text{adj } A$
 $|A| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1(1-0) - 1(0-0) = 1$
 $\therefore |A| = 1$
 $\text{adj } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
 $\therefore A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
 Similarly, $B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

09/04/20
XIII A + B

Answers

$$\text{and } 3X + 2Y = \begin{bmatrix} 2 & -3 \\ -1 & 5 \end{bmatrix}$$

Adding (i) & (ii), we get

$$5X + 3Y = \begin{bmatrix} 2 & 2 \\ 4 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 2 \\ 4 & 6 \end{bmatrix} \dots (iii)$$

$$\Rightarrow 5X + 3Y = \begin{bmatrix} 4 & 1 \\ 3 & 3 \end{bmatrix} \Rightarrow X + Y = \begin{bmatrix} 4 & 1 \\ 3 & 3 \end{bmatrix} \dots (iv)$$

Subtracting (i) from (iii), we get

$$(5X + 3Y) - (2X + 3Y) = \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 \\ 4 & 6 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 0 & -5 \\ -5 & 3 \end{bmatrix}$$

Finally, adding (iii) and (iv), we get

$$2X = \frac{1}{2} \begin{bmatrix} 4 & 1 \\ 3 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 4 & 3 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 6 & 1 \\ 2 & 4 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 3 & 0.5 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 4 & -5 \\ 5 & 5 \end{bmatrix} + \begin{bmatrix} 4 & -24 \\ 5 & 5 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 4 & -14.5 \\ 5 & 5 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2} \begin{bmatrix} 4 & -24 \\ 5 & 5 \end{bmatrix} \Rightarrow X = \begin{bmatrix} 2 & -12 \\ 2.5 & 2.5 \end{bmatrix}$$

Subtracting (ii) and (iv), we get $2Y = \frac{1}{2} \begin{bmatrix} 4 & 1 \\ 3 & 3 \end{bmatrix} - \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix}$

$$\Rightarrow 2Y = \begin{bmatrix} 4 & 1 \\ 3 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 5 & -5 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 8 & -2 \end{bmatrix}$$

$$\Rightarrow Y = \frac{1}{2} \begin{bmatrix} 4 & 6 \\ 8 & -2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$$

$$\text{Hence, } X = \begin{bmatrix} 2 & -12 \\ 2.5 & 2.5 \end{bmatrix}, Y = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$$

3. Find X, if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix}$

Soln.: We are given that, $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

$$\text{and } 2X + Y = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix}$$

Substituting the value of Y in (i), we have

$$2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -2 & -2 \\ -6 & -2 \end{bmatrix} \Rightarrow X = \frac{1}{2} \begin{bmatrix} -2 & -2 \\ -6 & -2 \end{bmatrix}$$

$$\text{Hence, } X = \begin{bmatrix} -1 & -1 \\ -3 & -1 \end{bmatrix}$$

9. Find x and y, if $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Soln.: We have, $2 \begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$$

$\Rightarrow 2 + y = 5$ and $2x + 2 = 8$ or $y = 3$ and $x = 3$
Hence, $x = 3$ and $y = 3$.

10. Solve the equation for x, y, z and t.

$$\text{if } 2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 2 \begin{bmatrix} 3 & 4 \\ 4 & 6 \end{bmatrix}$$

Soln.: $2 \begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 2 \begin{bmatrix} 3 & 4 \\ 4 & 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 8 & 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y+0 & 2t+6 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 8 & 12 \end{bmatrix}$$

Now, $2x + 3 = 6$ or $2x = 3$ or $x = \frac{3}{2}$
 $2z - 3 = 8$ or $2z = 11$ or $z = \frac{11}{2}$

$2y = 8$ or $y = 4$
 $2t + 6 = 12$ or $2t = 6$ or $t = 3$
Hence, $x = \frac{3}{2}$, $y = 4$, $z = \frac{11}{2}$ and $t = 3$.

11. If $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \end{bmatrix}$, find the values of x and y.

Soln.: We have, $x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x-y \\ 3x+y \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \end{bmatrix}$$

$\Rightarrow 2x - y = 10$... (i) and $3x + y = 8$... (ii)