Example 58 The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers. [NCERT]

Solution. Let the two required numbers be x and y and $x > y$.

Then, $x^2 - y^2 = 180$ and $y^2 = 8x$

On combining the above two equations, we get : $x^2 - 8x - 180 = 0$

Here, $a = 1$, $b = -8$, $c = -180$

$$\therefore \quad D = b^2 - 4ac = (-8)^2 - 4 \times 1 \times (-180) = 64 + 720 = 784 > 0$$

10|4

So, the real roots exist. Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{8 \pm \sqrt{784}}{2 \times 1} = \frac{8+28}{2}$$

$$\Rightarrow \quad x = \frac{8+28}{2} \quad \text{or} \quad x = \frac{8-28}{2} \quad \Rightarrow x = 18 \quad \text{or} \quad x = -10$$

Now, $x = 18 \Rightarrow y^2 = 8 \times 18 = 144 \quad \Rightarrow y = 12$ or $y = -12$

Also, $x = -10 \Rightarrow y^2 = 8 \times (-10) = -80$, which is not possible.

Hence, the required numbers are (18 and 12) or (18 and −12).

Q.4

Example 59 The sum of the reciprocals of Rehman's age (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Solution. Let the present age of Rehman $= x$ years

3 years ago Rehman's age $= x - 3$

5 years from now Rehman's age $= x + 5$

According to the question,

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3} \quad \Rightarrow \quad \frac{(x+5)+(x-3)}{(x-3)(x+5)} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{2x+2}{x^2+5x-3x-15} = \frac{1}{3} \quad \Rightarrow \quad x^2 + 2x - 15 = 3(2x+2) \quad \Rightarrow x^2 - 4x - 21 = 0$$

Here, $a = 1$, $b = -4$, $c = -21$

$$\therefore \quad D = b^2 - 4ac = (-4)^2 - 4 \times 1 \times (-21) = 16 + 84 = 100 > 0$$

So, the real roots exist. Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{4 \pm \sqrt{100}}{2 \times 1} = \frac{4 \pm 10}{2} = \frac{4+10}{2} \text{ or } \frac{4-10}{2}$$

$$\Rightarrow \quad x = 7 \quad \text{or} \quad x = -3$$

As the age of Rehman cannot be negative, $x \neq 3$. So $x = 7$.

Hence, the present age of Rehman $= 7$ years.

0|04|20 X - A+B

Example 3 Find the roots of the following equations : [NCERT]

(i) $x-\frac{1}{x}=3, x\neq 0$ (ii) $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}, x\neq -4,7.$

[CBSE D 08, 10C, 11]

Solution. (i) Given : $x-\frac{1}{x}=3 \quad\Rightarrow\quad \frac{x^2-1}{x}=3$

$\Rightarrow \quad x^2-1=3x \quad\Rightarrow\quad x^2-3x-1=0$

Here, $a=1,\ b=-3,\ c=-1$

$\therefore \quad D=b^2-4ac=(-3)^2-4\times 1(-1)=9+4=13>0$

$\Rightarrow \quad x=\frac{-b\pm\sqrt{D}}{2a}=\frac{-(-3)\pm\sqrt{13}}{2\times 1}=\frac{3\pm\sqrt{13}}{2}$

Hence, the roots of the given equation are $\frac{3+\sqrt{13}}{2}$ and $\frac{3-\sqrt{13}}{2}$.

(ii) Given : $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30},\quad x\neq -4,7$

$\Rightarrow \quad \frac{x-7-x-4}{(x+4)(x-7)}=\frac{11}{30} \quad\Rightarrow\quad \frac{-11}{(x+4)(x-7)}=\frac{11}{30}$

$\Rightarrow \quad x(x-7)+4(x-7)=-30$ [Dividing both sides by 11 and by cross-multiplication]

$\Rightarrow \quad x^2-7x+4x-28=-30 \quad\Rightarrow\quad x^2-3x+2=0$

Here, $a=1,\ b=-3,\ c=2$

$\therefore \quad D=b^2-4ac=(-3)^2-4\times 1\times 2=9-8=1>0$

So, the given equation has real roots which are given by

$$\alpha=\frac{-b+\sqrt{D}}{2a}=\frac{3+\sqrt{1}}{2\times 1}=\frac{3+1}{2}=2\ ; \qquad \beta=\frac{-b-\sqrt{D}}{2a}=\frac{3-1}{2}=\frac{2}{2}=1$$

Hence, the roots of the given equation are 1 and 2.

Q 5 **Example** In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product would have been 210. Find her marks in the two subjects. [NCERT]

Solution. Let marks secured by Shefali in Mathematics $= x$

Then marks secured by Shefali in English $= 30 - x$

According to the question,

2 marks more in Mathematics $\times$ 3 marks less in English $= 210$

$\Rightarrow \quad (x+2)\times[(30-x)-3] = 210 \quad \Rightarrow \quad (x+2)(27-x) = 210$

$\Rightarrow \quad 27x - x^2 + 54 - 2x = 210 \quad \Rightarrow \quad x^2 - 25x + 156 = 0$

Here, $a = 1, \quad b = -25, \quad c = 156$

$$\therefore \quad D = b^2 - 4ac = (-25)^2 - 4\times 156 = 625 - 624 = 1 > 0$$

So, the real roots exist. Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{25 \pm \sqrt{1}}{2\times 1} = \frac{25 \pm 1}{2} = 13 \text{ or } 12$$

$\Rightarrow$ Either Shefali got 12 marks in Mathematics and 18 marks in English or, she got 13 marks in Mathematics and 17 marks in English.

Q 6 **Example** The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

[NCERT ; CBSE D 09C]

Solution. Let the shorter side of rectangular field $= x$ m

$\therefore$ Larger side of the rectangular field $= (x+30)$ m and diagonal $= (x+60)$ m

$\therefore$ In right angled ΔABC,

$$AC^2 = AB^2 + BC^2$$

$$(x+60)^2 = (x+30)^2 + x^2$$

$$x^2 + 120x + 3600 = x^2 + 60x + 900 + x^2$$

$$\Rightarrow \quad x^2 - 60x - 2700 = 0$$

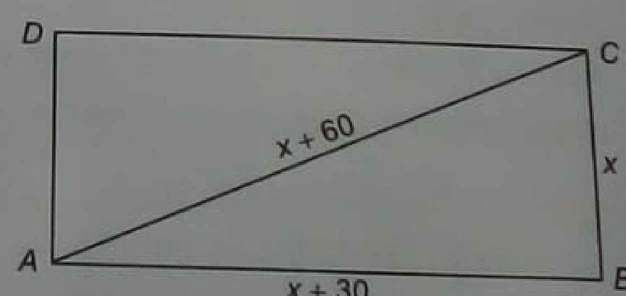

FIGURE 7.1

Here, $a = 1, \; b = -60, \; c = -2700$

$$\therefore \quad D = b^2 - 4ac = (-60)^2 - 4\times 1\times(-2700)$$

$$= 3600 + 10800 = 14400 > 0$$

So, the real roots exist. Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{60 \pm \sqrt{14400}}{2\times 1} = \frac{60 \pm 120}{2} \quad \Rightarrow \quad x = 90 \text{ or } x = -30$$

As the side of the rectangle cannot be negative, $x \neq -30$. So $x = 90$.

Hence, the shorter side of the rectangular field $=$ **90 m**

and the larger side of the rectangular field $= 90 + 30 =$ **120 m.**