$$\Rightarrow \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$$

$\Rightarrow 4k = 4 \Rightarrow k = 1$

Hence, $k = 1$.

18. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, then show that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.

Soln.: We know that,

$$\cos\alpha = \frac{1-\tan^2\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{1-t^2}{1+t^2}, \text{ where } \tan\frac{\alpha}{2} = t$$

$$\text{and } \sin\alpha = \frac{2\tan\left(\frac{\alpha}{2}\right)}{1+\tan^2\left(\frac{\alpha}{2}\right)} = \frac{2t}{1+t^2}$$

$$\text{Now, } I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

$$\text{and } I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix} = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}$$

$$\therefore (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}\begin{bmatrix} \frac{(1-t^2)}{(1+t^2)} & \frac{-2t}{(1+t^2)} \\ \frac{2t}{(1+t^2)} & \frac{(1-t^2)}{(1+t^2)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1-t^2}{1+t^2} + \frac{2t^2}{1+t^2} & \frac{-2t}{1+t^2} + \frac{t(1-t^2)}{1+t^2} \\ \frac{-t(1-t^2)}{1+t^2} + \frac{2t}{1+t^2} & \frac{2t^2}{(1+t^2)} + \frac{(1-t^2)}{(1+t^2)} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix} = (I + A)$$

$$\text{Hence, } (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = (I + A).$$

19. A trust fund has Rs. 30, 000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs. 30, 000 among the two types of bonds, if the trust fund obtains an annual total interest of :

(A) Rs. 1800 **(B) Rs. 2000**

Soln.: Let us take that the trust invests Rs. x at 5% p.a. and the trust invests Rs. $(30, 000 - x)$ at 7% p.a.

(A) So, $[x \quad 30,000 - x]\begin{bmatrix} 5\% \\ 7\% \end{bmatrix} = 1800$

$\Rightarrow \frac{5x}{100} + (30, 000 - x) \times \frac{7}{100} = 1800$

$\Rightarrow 5x + 2,10,000 - 7x = 1,80,000$

$\Rightarrow 2x = 2,10,000 - 1,80,000$

$\Rightarrow 2x = 30,000 \Rightarrow x = 15,000$

Hence, the trust invests Rs. 15,000 at 5% p.a. and Rs. $(30,000 - x)$ = Rs. $(30,000 - 15000)$ = Rs. 15,000 at 7% p.a.

(B) $[x \quad 30,000 - x]\begin{bmatrix} 5\% \\ 7\% \end{bmatrix} = 2000$

$\Rightarrow x \times \frac{5}{100} + (30, 000 - x) \times \frac{7}{100} = 2000$

$\Rightarrow 5x + 2,10,000 - 7x = 200000$

$\Rightarrow 2x = 2,10,000 - 2,00,000$

$\Rightarrow 2x = 10,000 \Rightarrow x = 5,000$

Hence, the trust invests Rs. 5,000 at 5% p.a. and Rs.(30,000 − 5,000) = Rs. 25,000 at 7% p.a.

20. The bookshop of a particular school has 10 do[zen] chemistry books, 8 dozen physics books, 10 dozen econom[ics] books. Their selling prices are Rs. 80, Rs. 60 and Rs. 40 [e]respectively. Find the total amount the bookshop will rec[eive] from selling all the books using matrix algebra.

Soln.: Number of Chemistry books = 10 dozen books = 120 b[ooks]

Number of Physics books = 8 dozen books = 96 books

Number of Economics books = 10 dozen books = 120 bo[oks]

$$\text{Now, } [120 \quad 96 \quad 120]\begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}$$

$= 120 \times 80 + 96 \times 60 + 120 \times 40$

$= 9,600 + 5,760 + 4,800 = 20,160$

Hence, total amount received = Rs. 20,160.

Assume X, Y, Z, W and P are matrices of order $2 \times n$, $2 \times p$, $n \times 3$ and $p \times k$, respectively. Choose the correct in questions 21 and 22.

21. The restriction on n, k and p so that $PY + WY$ defined are ____.

(A) $k = 3, p = n$ **(B)** k is arbitrary, $p =$

(C) p is arbitrary, $k = 3$ **(D)** $k = 2, p = 3$

Soln.: (A) Given : $X_{2\times n}, Y_{3\times k}, Z_{2\times p}, W_{n\times 3}, P_{p\times k}$

Now, $PY + WY = P_{p\times k} \times Y_{3\times k} + W_{n\times 3} \times Y_{3\times k}$

Clearly, $k = 3$ and $p = n$

22. If $n = p$, then the order of the matrix $7X - 5Z$ i[s]

(A) $p \times 2$ **(B)** $2 \times n$

(C) $n \times 3$ **(D)** $p \times n$

order n = p and 2×n

 Q15 XII 10/4/20 Ex-3.2

Soln:

$$\Rightarrow A^2 = A\cdot A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times2+0\times2+1\times1 & 2\times0+0\times1+1\times(-1) & 2\times1+0\times3+1\times0 \\ 2\times2+1\times2+3\times1 & 2\times0+1\times1+3\times(-1) & 2\times1+1\times3+3\times0 \\ 1\times2+2\times(-1)+0\times1 & 1\times0+(-1)\times1+0\times(-1) & 1\times1+(-1)\times3+0\times0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix}$$

$$6I = 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\therefore A^2-5A+6I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

16. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = O$.

Soln.: We are given that, $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

$$\Rightarrow A^2 = A\cdot A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+0\times0+2\times2 & 0\times1+0\times2+0\times2 & 1\times2+0\times1+2\times3 \\ 0\times1+2\times0+1\times2 & 0\times0+2\times2+1\times0 & 0\times2+2\times1+3\times1 \\ 2\times1+0\times0+3\times2 & 2\times0+0\times2+3\times0 & 2\times2+0\times1+3\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

Now, $A^3 = A^2\cdot A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 5\times1+0\times0+2\times8 & 5\times0+0\times2+8\times0 & 5\times2+0\times1+8\times3 \\ 2\times1+4\times0+5\times2 & 2\times0+4\times2+5\times0 & 2\times2+4\times1+5\times3 \\ 8\times1+0\times0+13\times2 & 8\times0+0\times2+13\times0 & 2\times8+1\times0+13\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}.$$

Now L.H.S. $= A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21-30 & 0-0 & 34-48 \\ 12-12 & 8-24 & 23-30 \\ 34-48 & 0-0 & 55-78 \end{bmatrix} + \begin{bmatrix} 7+2 & 0+0 & 14+0 \\ 0+0 & 14+2 & 7+0 \\ 14+0 & 0+0 & 21+2 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 0 & -14 \\ 0 & -16 & -7 \\ -14 & 0 & -23 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 14 \\ 0 & 16 & 7 \\ 14 & 0 & 23 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0 = \text{R.H.S.}$$

Hence, proved.

17. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$.

Soln.: We are given that, $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Also, $A^2 = kA - 2I$

Substituting the values of A and I from above, we get

$$\Rightarrow \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = k\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3\times3+(-2)\times4 & 3\times(-2)+(-2)\times(-2) \\ 4\times3+(-2)\times4 & 4\times(-2)+(-2)\times(-2) \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} =$$

$4k = 4 \Rightarrow k = 1$