

Q9:

$$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$

$$\frac{(x-10)+x}{x(x-10)} = \frac{8}{75}$$

$$\Rightarrow \frac{2x-10}{x^2-10x} = \frac{8}{75}$$

$$\Rightarrow 8x^2 - 230x + 750 = 0$$

$$8x^2 - 80x = 150x - 750$$

$$4x^2 - 115x + 375 = 0$$

$$4x^2 - 100x - 15x + 375 = 0$$

$$4x(x-25) - 15(x-25) = 0$$

$$(x-25)(4x-15) = 0$$

$$\Rightarrow x-25=0 \text{ or } 4x-15=0$$

$$\Rightarrow x=25 \text{ or } x=\frac{15}{4}$$

Now, $x=\frac{15}{4} \Rightarrow x-10 < 0$

i.e., the time taken by the larger pipe is negative, which is not possible. So $x=25$.

Hence, the time taken by the smaller tap to fill the tank = 25 h

and the time taken by the larger tap to fill the tank = $25-10=15$ h.

~~Example 60. Out of a number of some kind, one fourth of the number are moving about~~

X 1119

Also, sum of areas = 468 m^2

[Given]

$$\Rightarrow x^2 + y^2 = 468 \quad \Rightarrow x^2 + (x-6)^2 = 468$$

[: $y = x-6$]

$$\Rightarrow x^2 + x^2 - 12x + 36 = 468$$

$$\Rightarrow 2x^2 - 12x + 36 - 468 = 0 \quad \Rightarrow x^2 - 6x - 216 = 0$$

Here, $a=1$, $b=-6$, $c=-216$

$$\therefore D = b^2 - 4ac = (-6)^2 - 4 \times 1 \times (-216) = 36 + 864 = 900 > 0$$

So, the real roots exist. Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{6 \pm \sqrt{900}}{2 \times 1} = \frac{6 \pm 30}{2} = 18 \text{ or } -12$$

As the side of a square cannot be negative, $x \neq -12$. So $x=18 \Rightarrow y=18-6=12$

Hence, the sides of the two squares are 18 m and 12 m respectively.

Example 68 Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution. Let the smaller tap fill the tank in x hours.
Then the larger fills the tank in $(x-10)$ hours.

Time taken by both taps together to fill the tank $= 9\frac{3}{8}$ h $= \frac{75}{8}$ h

Part of tank filled by the smaller tap in 1 h $= \frac{1}{x}$

Part of tank filled by the larger tap in 1 h $= \frac{1}{x-10}$

Part of tank filled by both taps together in 1 h $= \frac{8}{75}$

Time taken at increased speed = $\frac{360}{x+5}$ h

Time difference in the two cases = 1 h

$$\frac{360}{x} - \frac{360}{x+5} = 1 \Rightarrow \frac{360x + 1800 - 360x}{x(x+5)} = 1$$

[Given]

$$\Rightarrow x^2 + 5x - 1800 = 0$$

Here, $a=1$, $b=5$, $c=-1800$

$$\therefore D = b^2 - 4ac = 5^2 - 4 \times 1 \times (-1800) = 25 + 7200 = 7225 > 0$$

So, the real roots exist. Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm \sqrt{7225}}{2 \times 1} = \frac{-5 \pm 85}{2} = 40 \text{ or } -45$$

As speed of the train cannot be negative, $x \neq -45$. So $x = 40$.

Hence, the speed of the train = 40 km/h.

Example An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains. [NCERT]

Solution. Let the speed of the passenger train = x km/h

Then, the speed of the express train = $(x+11)$ km/h

Time taken by passenger train to travel 132 km = $\frac{132}{x}$ h

Time taken by express train to travel 132 km = $\frac{132}{x+11}$ h

According to the question,

$$\frac{132}{x} - \frac{132}{x+11} = 1 \Rightarrow \frac{132x + 1452 - 132x}{x(x+11)} = 1$$

$$\Rightarrow 1452 = x^2 + 11x \Rightarrow x^2 + 11x - 1452 = 0$$

Here, $a=1$, $b=11$, $c=-1452$

$$\therefore D = b^2 - 4ac = 11^2 - 4 \times 1 \times (-1452) = 121 + 5808 = 5929 > 0$$

So, the real roots exist. Using the quadratic formula,

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-11 \pm \sqrt{5929}}{2 \times 1} = \frac{-11 \pm 77}{2} = 33 \text{ or } -44$$

As the speed of a train cannot be negative, $x \neq -44$. So $x = 33$.

Hence, the speed of the passenger train = 33 km/h

and the speed of the express train = $33 + 11 = 44$ km/h.

Example Sum of the areas of two squares is 468 m^2 . If the difference of their perimeters is 24 m, find the sides of the two squares. [NCERT]

Solution. Let the sides of two squares be x m and y m such that $x > y$.

Then their areas will be $(x^2) \text{ m}^2$ and $(y^2) \text{ m}^2$ respectively, and their perimeters will be $(4x) \text{ m}$ and $(4y) \text{ m}$ respectively.

Now, difference of perimeters = 24 m

[Given]

$$\Rightarrow 4x - 4y = 24 \Rightarrow x - y = 6 \Rightarrow y = x - 6$$