

As $D > 0$, the given equation has two distinct real roots.

Example 72 Find the nature of the roots of the following quadratic equations. If the real roots exist, find them: [NCERT]

(i) $2x^2 - 3x + 5 = 0$ (ii) $3x^2 - 4\sqrt{3}x + 4 = 0$ (iii) $2x^2 - 6x + 3 = 0$

Solution. (i) Given: $2x^2 - 3x + 5 = 0$. Here, $a = 2$, $b = -3$, $c = 5$
 $\therefore D = b^2 - 4ac = (-3)^2 - 4 \times 2 \times 5 = 9 - 40 = -31 < 0$

As $D < 0$, the given equation has no real roots.

(ii) Given: $3x^2 - 4\sqrt{3}x + 4 = 0$. Here, $a = 3$, $b = -4\sqrt{3}$, $c = 4$
 $\therefore D = b^2 - 4ac = (-4\sqrt{3})^2 - 4 \times 3 \times 4 = 48 - 48 = 0$

As $D = 0$, the given equation has real and equal roots.

Each root $= \frac{-b}{2a} = \frac{-(-4\sqrt{3})}{2 \times 3} = \frac{2\sqrt{3}}{3}$

Hence, the roots of the given equation are $\frac{2\sqrt{3}}{3}$ and $\frac{2\sqrt{3}}{3}$.

(iii) Given: $2x^2 - 6x + 3 = 0$. Here, $a = 2$, $b = -6$, $c = 3$
 $\therefore D = b^2 - 4ac = (-6)^2 - 4 \times 2 \times 3 = 36 - 24 = 12 > 0$

As $D > 0$, the given equation has real and distinct roots.

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-6) \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}$$

Hence, the roots of the equation are: $\frac{3 + \sqrt{3}}{2}$ and $\frac{3 - \sqrt{3}}{2}$.

Example 73 Find the values of k for each of the following quadratic equations, so that they have two equal roots: [NCERT ; CBSE OD 12, D 1]

(i) $2x^2 + kx + 3 = 0$ (ii) $kx(x - 2) + 6 = 0$

Solution. (i) Given: $2x^2 + kx + 3 = 0$. Here, $a = 2$, $b = k$, $c = 3$

$\therefore D = b^2 - 4ac = k^2 - 4 \times 2 \times 3 = k^2 - 24$

For equal roots, $D = 0 \Rightarrow k^2 - 24 = 0 \Rightarrow k = \pm 2\sqrt{6}$

(ii) Given: $kx(x - 2) + 6 = 0$

$kx^2 - 2kx + 6 = 0$

Here, $a = k$, $b = -2k$, $c = 6$

$\therefore D = b^2 - 4ac = 4k^2 - 4 \times k \times 6 = 4k(k - 6)$

For equal roots, $D = 0 \Rightarrow 4k(k - 6) = 0 \Rightarrow k = 0$ or $k = 6$

But the given equation will not be quadratic for $k = 0$, so $k \neq 0$.

Hence, $k = 6$.

Example 73 For what value of k , are the roots of the quadratic equation $(k + 4)x^2 + (k + 1)x + 1 = 0$ equal? [CBSE OD 12, D 1]

13/4
X tk

Q1
EX
4.4
X

~~8~~ 13/4

$$\Rightarrow x(40-x) = 400$$

$$\Rightarrow x^2 - 40x + 400 = 0$$

$$\text{Here, } a=1, b=-40, c=400$$

$$\therefore D = b^2 - 4ac = (-40)^2 - 4 \times 1 \times 400 = 1600 - 1600 = 0$$

So, the given equation has real and equal roots. Hence, it is possible to design a rectangular park within given conditions,

$$\text{Each root, } x = -\frac{b}{2a} = -\frac{-40}{2 \times 1} = 20$$

$$\therefore \text{Length} = 20 \text{ m and breadth} = 40 - 20 = 20 \text{ m}$$

Clearly, the rectangular park has a square shape.

Example Is the following situation possible? If so, determine their present ages. The sum of the ages of a mother and her daughter is 20 years. Four years ago, the product of their ages in years was 48. [NCERT]

Solution. Let the present age of the daughter = x years

Then, the present age of the mother = $(20 - x)$ years

4 years ago daughter's age = $(x - 4)$ years

4 years ago mother's age = $20 - x - 4 = 16 - x$

According to the question,

$$(x-4)(16-x) = 48 \quad \Rightarrow 16x - x^2 - 64 + 4x = 48 \quad \Rightarrow x^2 - 20x + 112 = 0$$

Here, $a=1, b=-20, c=112$

$\therefore D = b^2 - 4ac = (-20)^2 - 4 \times 1 \times 112 = 400 - 448 = -48 < 0$, no real values of x exist.

Hence, the given situation is *not possible*.

Example 86 In each of the following, determine value(s) of p for which the given quadratic

X 13/4

Q3 Example 83 Is it possible to design a rectangular mango grove whose length is twice its breadth, and area is 800 m^2 ? If so, find its length and breadth. [NCERT]

Solution. Let the breadth of rectangular grove = x metres.

Then the length of the rectangular grove = $2x$ metres

$$\therefore \text{Area} = 2x \times x = 800 \text{ m}^2 \quad \Rightarrow x^2 = 400 \quad \Rightarrow x = \pm 20$$

As breadth cannot be negative, $x \neq -20$. So, $x = +20$.

The real value of x implies, it is possible to design the desired grove.

Hence, its breadth = 20 m and length = 40 m .

Q4 Example 84 Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2 ? If so, find its length and breadth. [NCERT]

Solution. Let length of the rectangular park = x metres

and breadth of the rectangular park = y metres

$$\therefore \text{Its perimeter} = 2(x + y) = 80 \text{ m} \quad \Rightarrow x + y = 40$$

$$\text{Its area} = xy = 400 \text{ m}^2$$