XII 18/04/20

9. $[x \quad -5 \quad -1]\begin{bmatrix} x+2 \\ 9 \\ 2x+3 \end{bmatrix} = O$

$\Rightarrow x(x+2) - 45 - 2x - 3 = 0 \Rightarrow x^2 - 48 = 0$

$\Rightarrow x = \pm 4\sqrt{3}$.

10. A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated as :

Market	Products		
I	10,000	2,000	18,000
II	6,000	20,000	8,000

(a) If unit sale prices of x, y and z are Rs 2.50, Rs 1.50 and Rs 1.00, respectively, find the total revenue in each market with the help of matrix algebra.

(b) If the unit costs of the above three commodities are Rs 2.00, Rs 1.00 and 50 paise respectively. Find the gross profit.

Soln.: Let quantity matrix be $A = \begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix}$

(a) Selling Price $B = \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$

Now, Total Selling Price,

$$AB = \begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix}\begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix}$$

$$= \begin{bmatrix} 10,000 \times 2.50 + 2,000 \times 1.50 + 18,000 \times 1 \\ 6,000 \times 2.50 + 20,000 \times 1.50 + 8,000 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} 25,000 + 3,000 + 18,000 \\ 15,000 + 30,000 + 8,000 \end{bmatrix} = \begin{bmatrix} 46,000 \\ 53,000 \end{bmatrix}$$

Total revenue in market I = Rs. 46,000.

Total revenue in market II = Rs. 53,000.

(b) Now, cost price $= \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix}$

$$\text{Total cost price} = \begin{bmatrix} 10,000 & 2,000 & 18,000 \\ 6,000 & 20,000 & 8,000 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 10,000 \times 2 + 2,000 \times 1 + 18,000 \times 0.5 \\ 6,000 \times 2 + 20,000 \times 1 + 8,000 \times 0.5 \end{bmatrix} = \begin{bmatrix} 31,000 \\ 36,000 \end{bmatrix}$$

Total cost price = 31000 + 36000 = Rs. 67,000.

Total selling price = 46000 + 53000 = Rs. 99,000

Profit = S.P. – C.P. = 99,000 – 67,000 = Rs. 32,000.

11. Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$.

Soln.: $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

We can say that X is a 2×2 matrix.

Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\Rightarrow a + 4b = -7$ .... (i) and $c + 4d = 2$ ...(ii)

$2a + 5b = -8$ .... (iii) and $2c + 5d = 4$ ...(iv)

Solving (i) and (iii), we get $a = 1, b = -2$

Solving (ii) and (iv), we get $c = 2, d = 0$

Hence, $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$.

12. If A and B are square matrices of the same order such that $AB = BA$, then prove by induction that $AB^n = B^nA$. Further, prove that $(AB)^n = A^nB^n$ for all $n \in N$.

Soln.: Given $AB = BA$, ...(

To prove

(1) $AB^n = B^nA$ and (2) $(AB)^n = A^n.B^n \ \forall n \in N$ ...(

We will prove it by mathematical induction.

(1) Given that $AB = BA$

We have to prove $AB^n = B^nA$

For $n = 1$, $AB^1 = B^1A$ [from (

$\Rightarrow AB = BA$, which is true

Let it be true for $n = m$, i.e., $AB^m = B^mA$ ...

Then, for $n = m + 1$,

$AB^{m+1} = A(B^mB) = (AB^m)B = (B^mA)B$ [using (

$= B^m(AB) = B^m(BA)$ [using

$= (B^mB)A = B^{m+1}A$. So, it is true for $n = m + 1$

$\therefore AB^n = B^nA$.

(2) For $n = 1$, $(AB)^1 = A^1B^1$

$\Rightarrow AB = BA$ which is true for $n = 1$

Let (i) be true for a positive integer $n = m$.

i.e., $(AB)^m = A^mB^m$

then for $n = m + 1$, $(AB)^{m+1} = (AB)^m(AB)$

$= (A^mB^m)(AB)$ (from (iv))

$= A^m(B^mA)B$

$= A^m(AB^m)B$ [$AB^n = B^nA \ \forall n$, whenever AB

$= (A^mA)(B^mB) = A^{m+1}B^{m+1}$

So, it holds for $n = m + 1$

Hence, $(AB)^n = A^nB^n \ \forall n \in N$.

Choose the correct answer in the following question

13. If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

XII 08/04/20

(A) $1+\alpha^2+\beta\gamma=0$ (B) $1-\alpha^2+\beta\gamma=0$

(C) $1-\alpha^2-\beta\gamma=0$ (D) $1+\alpha^2-\beta\gamma=0$

Soln.: (C) Given $A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$

Now, $A^2=I$

$$\Rightarrow \begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}\alpha^2+\beta\gamma & \alpha\beta-\alpha\beta\\ \gamma\alpha-\alpha\gamma & \gamma\beta+\alpha^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}\alpha^2+\beta\gamma & 0\\ 0 & \gamma\beta+\alpha^2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \alpha^2+\beta\gamma=1 \Rightarrow 1-\alpha^2-\gamma\beta=0$$

14. If the matrix A is both symmetric and skew symmetric, then

(A) A is a diagonal matrix (B) A is a zero matrix

(C) A is a square matrix (D) None of these

Soln.: (B) Consider the matrix A.

Clearly, $A'=A$ and $A'=-A$

$\therefore\ A=-A \Rightarrow 2A=0 \Rightarrow A=0$

$\therefore\ A$ is a zero matrix.

15. If A is a square matrix such that $A^2=A$, then $(I+A)^3-7A$ is equal to

(A) A (B) $I-A$

(C) I (D) $3A$

Soln.: (C) We are given that $A^2=A$

Now, $(I+A)^3-7A=I^3+A^3+3IA\,(I+A)-7A$

$=I+A^2+3A\,(I+A)-7A=I+A^2+3A+3A^2-7A$

$=I+4A^2-4A=I+4A-4A=I$