X (5.1) 2/4

(xv) $1^2, 5^2, 7^2, 73, \ldots$

Here, $a_2 - a_1 = 5^2 - 1^2 = 25 - 1 = 24$; $a_3 - a_2 = 7^2 - 5^2 = 49 - 25 = 24$

$a_4 - a_3 = 73 - 49 = 24$

$\Rightarrow \quad a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

$\Rightarrow \quad a_{k+1} - a_k$ is the same for different values of k

Hence, the given list of numbers forms an AP with common difference, $d = 24$

The next three terms are :

$a_5 = 73 + 24 = 97, \quad a_6 = 97 + 24 = 121, \quad a_7 = 121 + 24 = 145.$

Example 3 For the following APs, write the first term and the common difference :

(*i*) 3, 1, −1, −3,... (*ii*) −5, −1, 3, 7,... (*iii*) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots$ (*iv*) 0.6, 1.7, 2.8, 3.9,... [NCERT]

Solution.

	AP	First term	Common difference
1.	3, 1, −1, −3,...	$a = 3$	$d = 1 - 3 = -2$
2.	−5, −1, 3, 7,...	$a = -5$	$d = -1 - (-5) = 4$
3.	$\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots$	$a = \frac{1}{3}$	$d = \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$
4.	0.6, 1.7, 2.8, 3.9,...	$a = 0.6$	$d = 1.7 - 0.6 = 1.1$

Example 4 Which of the following are APs ? If they form an AP, find the common difference *d* and write *three* more terms :

(*i*) 2, 4, 8, 16,... (*ii*) $2, \frac{5}{2}, 3, \frac{7}{2}, \ldots$ (*iii*) −1.2, −3.2, −5.2, −7.2, ...

(*iv*) −10, −6, −2, 2,... (*v*) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, \ldots$

(*vi*) 0.2, 0.22, 0.222,... (*vii*) 0, −4, −8, −12,...

(*viii*) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots$ (*ix*) 1, 3, 9, 27,...

(*x*) $a, 2a, 3a, 4a, \ldots$ (*xi*) $a, a^2, a^3, a^4, \ldots$ (*xii*) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, \ldots$ [CBSE D 09 C]

(*xiii*) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \ldots$ (*xiv*) $1^2, 3^2, 5^2, 7^2, \ldots$ (*xv*) $1^2, 5^2, 7^2, 73, \ldots$ [NCERT]

Solution. (*i*) 2, 4, 8, 16,...

Here, $a_2 - a_1 = 4 - 2 = 2$; $a_3 - a_2 = 8 - 4 = 4$

$\Rightarrow$ $a_2 - a_1 \neq a_3 - a_2$

Hence, the given list of numbers **does not form an AP.**

(*ii*) $2, \frac{5}{2}, 3, \frac{7}{2}$

Here, $a_2 - a_1 = \frac{5}{2} - 2 = \frac{5-4}{2} = \frac{1}{2}$; $a_3 - a_2 = 3 - \frac{5}{2} = \frac{6-5}{2} = \frac{1}{2}$; $a_4 - a_3 = \frac{7}{2} - 3 = \frac{7-6}{2} = \frac{1}{2}$

$\Rightarrow$ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

(*viii*) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \ldots$

Here, $a_2 - a_1 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$; $a_3 - a_2 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$

$a_4 - a_3 = -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$

$\Rightarrow$ $a_{k+1} - a_k$ is same in each case

Hence, the given list of numbers forms an AP with common difference, $d = 0$

The next three terms are : $a_5 = -\frac{1}{2}$, $a_6 = -\frac{1}{2}$, $a_7 = -\frac{1}{2}$.

(*ix*) 1, 3, 9, 27,...

Here, $a_2 - a_1 = 3 - 1 = 2$; $a_3 - a_2 = 9 - 3 = 6$

$\Rightarrow$ $a_2 - a_1 \neq a_3 - a_2$

Hence, the given list of numbers **does not form an AP**.

(*x*) a, $2a$, $3a$, $4a$,...

Here, $a_2 - a_1 = 2a - a = a$; $a_3 - a_2 = 3a - 2a = a$; $a_4 - a_3 = 4a - 3a = a$

$\Rightarrow$ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

$\Rightarrow$ $a_{k+1} - a_k$ is same in each case

Hence, the given list of numbers forms an AP with common difference, $d = a$

The next three terms are : $a_5 = a_4 + d = 4a + a = 5a$, $a_6 = 6a$, $a_7 = 7a$.

(*xi*) a, a^2, a^3, a^4,....

Here, $a_2 - a_1 = a^2 - a = a(a-1)$; $a_3 - a_2 = a^3 - a^2 = a^2(a-1)$

$\Rightarrow$ $a_2 - a_1 \neq a_3 - a_2$

Hence, the given list of numbers **does not form an AP**.

(*xii*) $\sqrt{2}$, $\sqrt{8}$, $\sqrt{18}$, $\sqrt{32}$,...

Here, $a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$; $a_3 - a_2 = \sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$a_4 - a_3 = \sqrt{32} - \sqrt{18} = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

$\Rightarrow$ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

$\Rightarrow$ $a_{k+1} - a_k$ is same in each case

Hence, the given list of numbers forms an AP with common difference, $d = \sqrt{2}$

The next three terms are :

$a_5 = a_4 + d = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$, $a_6 = a_5 + d = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$, $a_7 = a_6 + d = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2}$

(*xiii*) $\sqrt{3}$, $\sqrt{6}$, $\sqrt{9}$, $\sqrt{12}$,...

Here, $a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)$; $a_3 - a_2 = \sqrt{9} - \sqrt{6} = \sqrt{3}(\sqrt{3} - \sqrt{2})$

$\Rightarrow$ $a_2 - a_1 \neq a_3 - a_2$

Hence, the given list of numbers **does not form an AP**.

(*xiv*) $1^2, 3^2, 5^2, 7^2, \ldots$

Here, $a_2 - a_1 = 3^2 - 1^1 = 9 - 1 = 8$; $a_3 - a_2 = 5^2 - 3^2 = 25 - 9 = 16$

$\Rightarrow$ $a_2 - a_1 \neq a_3 - a_2$

Hence, the given list of numbers **does not form an AP**.

$\Rightarrow$ $a_{k+1} - a_k$ is same in each case

Hence, the given list of numbers forms an AP with common difference, $d = \frac{1}{2}$

The next three terms are :

$$a_5 = a_4 + d = \frac{7}{2} + \frac{1}{2} = 4, \quad a_6 = a_5 + d = 4 + \frac{1}{2} = \frac{9}{2}, \quad a_7 = a_6 + d = \frac{9}{2} + \frac{1}{2} = 5.$$

(iii) $-1.2, -3.2, -5.2, -7.2, \ldots$

Here, $a_2 - a_1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2$; $a_3 - a_2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2$

$a_4 - a_3 = -7.2 - (-5.2) = -7.5 + 5.2 = -2$

$\Rightarrow$ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

$\Rightarrow$ $a_{k+1} - a_k$ is same in each case

Hence, the given list of numbers forms an AP with common difference $d = -2$

The next three terms are :

$$a_5 = a_4 + d = -7.2 + (-2) = -9.2, \quad a_6 = -9.2 + (-2) = -11.2, \quad a_7 = -11.2 + (-2) = -13.2.$$

(iv) $-10, -6, -2, 2 \ldots$

Here, $a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$; $a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$

$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

$\Rightarrow$ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

$\Rightarrow$ $a_{k+1} - a_k$ is same in each case

Hence, the given list of numbers forms an AP with common difference, $d = 4$

The next three terms are :

$$a_5 = a_4 + d = 2 + 4 = 6, \quad a_6 = 6 + 4 = 10, \quad a_7 = 10 + 4 = 14.$$

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \ldots$

Here, $a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$; $a_3 - a_2 = (3 + 2\sqrt{2}) - (3 + \sqrt{2}) = \sqrt{2}$

$a_4 - a_3 = (3 + 3\sqrt{2}) - (3 + 2\sqrt{2}) = \sqrt{2}$

$\Rightarrow$ $a_{k+1} - a_k$ is same in each case

Hence, the given list of numbers forms an AP with common difference, $d = \sqrt{2}$

The next three terms are :

$$a_5 = a_4 + \sqrt{2} = 3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2} \quad a_6 = a_5 + \sqrt{2} = 3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$$

$$a_7 = a_6 + \sqrt{2} = 3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}.$$

(vi) 0.2, 0.22, 0.222, 0.2222, ...

Here, $a_2 - a_1 = 0.22 - 0.2 = 0.02$; $a_3 - a_2 = 0.222 - 0.22 = 0.002$

$\Rightarrow$ $a_2 - a_1 \neq a_3 - a_2$

The given list of numbers **does not form an AP**.

(vii) $0, -4, -8, -12, \ldots$

Here, $a_2 - a_1 = -4 - 0 = -4$; $a_3 - a_2 = -8 - (-4) = -8 + 4 = -4$

$a_4 - a_3 = -12 - (-8) = -12 + 8 = -4$

$\Rightarrow$ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

$\Rightarrow$ $a_{k+1} - a_k$ is same in each case

Hence, the given list of numbers forms an AP with common difference, $d = -4$

The next three terms are :

$$a_5 = a_4 + d = -12 + (-4) = -16, \quad a_6 = -16 + (-4) = -20, \quad a_7 = -20 + -(4) = -24.$$