$\Rightarrow$ $Y \in P(A)$ $[\because P(A) = P(B)]$

$\Rightarrow$ $Y \subset A \Rightarrow Y \in A.$

Thus, $y \in B \Rightarrow y \in A$

$\therefore$ $B \subset A.$

From (1) and (2), we get

$A = B.$

7. Is it true that, for any sets A and B, $P(A) \cup P(B) = P(A \cup B)$? Justify your answer.

Sol. No.

Let $A = \{a\}$, $B = \{b\}$ and $A \cup B = \{a, b\}$.

$\therefore$ $P(A) = \{\phi, \{a\}\}, P(B) = \{\phi, \{b\}\}$

and $P(A \cup B) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$ ... (1)

and $P(A) \cup P(B) = \{\phi, \{a\}, \{b\}\}$ ... (2)

From (1) and (2), we have :

$P(A \cup B) \neq P(A) \cup P(B).$

8. Show that for any sets A and B, $A = (A \cap B) \cup (A - B)$ and $A \cup (B - A) = A \cup B$.

Sol. $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$ $[\because A - B = A \cap B']$

$= A \cap (B \cup B')$ [*Distributive law*]

$= A \cap X$, where X is universal set

$= A.$

$A \cup (B - A) = A \cup (B \cap A')$ $[\because B - A = B \cap A']$

$= (A \cup B) \cap (A \cup A')$ [*Distributive law*]

$= (A \cup B) \cap X,$

where $X = A \cup A'$ is universal set

$= A \cup B$ $[\because A \cup B \subset X]$

9. Using properties of sets, show that

(*i*) $A \cup (A \cap B) = A$ **(*ii*) $A \cap (A \cup B) = A$.**

Sol. (*i*) $A \cup (A \cap B) = (A \cup A) \cap (A \cup B)$ [*Distributive law*]

$= A \cap (A \cup B)$ $[\because A \cup A = A]$

$= A$ $[\because A \subseteq A \cup B]$

(*ii*) $A \cap (A \cup B) = (A \cap A) \cup (A \cap B)$

$= A \cup (A \cap B)$

$= A$ $[\because A \cap B \subset A]$

10. Show that $A \cap B = A \cap C$ need not imply $B = C$.

Sol. With the help of an example, we may try to establish it.

Let $A = \{1, 2\}$, $B = \{1, 3\}$ and $C = \{1, 4\}$.

Now, $A \cap B = \{1, 2\} \cap \{1, 3\} = \{1\}$

and $A \cap C = \{1, 2\} \cap \{1, 4\} = \{1\}.$

$\therefore$ $A \cap B = A \cap C,$

still $B \neq C.$

MISCELLANEOUS EXERCISE

1. Decide, among the following sets, which are the subsets of one and another : A = $\{x : x \in R$ and x satisfy $x^2 - 8x + 12 = 0\}$, B = {2, 4, 6}, C = {2, 4, 6, 8, ...}, D = {6}.

Sol. We have : A = {2, 6}, B = {2, 4, 6}, C = {2, 4, 6, 8, ...} and D = {6}.

Clearly, every element of A is in B and C.

$\therefore A \subset B$ and $A \subset C$.

Again, every element of B is in C.

$\therefore B \subset C$.

Also, every element of D is in A, B and C.

$\therefore D \subset A, D \subset B$ and $D \subset C$.

2. In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.

(*i*) If $x \in A$ and $A \in B$, then $x \in B$.

(*ii*) If $A \subset B$ and $B \in C$, then $A \in C$.

(*iii*) If $A \subset B$ and $B \subset C$, then $A \subset C$.

(*iv*) If $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$.

(*v*) If $x \in A$ and $A \not\subset B$, then $x \in B$.

(*vi*) If $A \subset B$ and $x \notin B$, then $x \notin A$.

Sol. (*i*) **False.** Let A = {1}, B = {{1}, 2}

Clearly, $1 \in A$ and $A \in B$ but $1 \notin B$.

So, $x \in A$ and $A \in B$ need not imply that $x \in B$.

(*ii*) **False.** Let A = {1}, B = {1, 2} and C = {{1, 2}, 3}

Clearly, $A \subset B$ and $B \in C$ but $A \notin C$

Thus, $A \subset B$ and $B \in C$ need not imply that $A \in C$.

(*iii*) **True.** Let $x \in A$. Then,

$A \subset B \Rightarrow x \in B \Rightarrow x \in C$.

Thus, $x \in A \Rightarrow x \in C$ for all $x \in A \Rightarrow A \subset C$

Hence, $A \subset B$ and $B \subset C \Rightarrow A \subset C$.

(*iv*) **False.** Let A = {1, 2}, B = {2, 3} and C = {1, 2, 5}

Then, $A \not\subset B$ and $B \not\subset C$. But $A \subset C$.

Thus, $A \not\subset B$ and $B \not\subset C$ need not imply that $A \not\subset C$.

(*v*) **False.** Let A = {1, 2} and B = {2, 3, 4, 5}

Clearly, $1 \in A$ and $A \not\subset B$ but $1 \notin B$.

Thus, $x \in A$ and $A \not\subset B$ need not imply that $x \in B$.

(*vi*) **True.** Let $A \subset B$. Then, clearly $x \in A \Rightarrow x \in B \Leftrightarrow x \notin B \Rightarrow x \notin A$.

3. Let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$. Show that $B = C$.

Sol. We have $A \cup B = A \cup C$.

$\Rightarrow \quad (A \cup B) \cap C = (A \cup C) \cap C$

$\Rightarrow \quad (A \cap C) \cup (B \cap C) = C \quad [\because (A \cup C) \cap C = C]$

$\Rightarrow \quad (A \cap B) \cup (B \cap C) = C \quad \ldots (1) \; [\because A \cap C = A \cap B]$

Again, $A \cup B = A \cup C$

$\Rightarrow \quad (A \cup B) \cap B = (A \cup C) \cap B$

$\Rightarrow \quad B = (A \cap B) \cup (C \cap B) \quad [\because (A \cup B) \cap B = B]$

$\Rightarrow \quad B = (A \cap B) \cup (B \cap C) \quad \ldots (2)$

From (1) and (2), we get

$B = C$.

4. Show that the following four conditions are equivalent (i) $A \subset B$ (ii) $A - B = \phi$ (iii) $A \cup B = B$ (iv) $A \cap B = A$

Sol. (*i*) $\Leftrightarrow$ (*ii*)

$A \subset B \Leftrightarrow$ All elements of A are in B $\Leftrightarrow A - B = \phi$.

(*ii*) $\Leftrightarrow$ (*iii*)

$A - B = \phi \Leftrightarrow$ All the elements of A are in B $\Leftrightarrow A \cup B = B$.

(*iii*) $\Leftrightarrow$ (*v*)

$A \cup B = B \Leftrightarrow$ All the elements of A are in B.

$\Leftrightarrow$ All the elements of A are common in A and B $\Leftrightarrow A \cap B = A$

Thus, all the four given conditions are equivalent.

5. Show that if $A \subset B$, then $C - B \subset C - A$.

Sol. $x \in C - B \Rightarrow x \in C$ but $x \notin B$

$\Rightarrow \quad x \in C$ but $x \notin A$

$\Rightarrow \quad x \in C - A$

$\therefore \quad C - B \subset C - A$.

6. Assume that $P(A) = P(B)$. Prove that $A = B$.

Sol. Let x be an arbitrary element of A. Then, there exists a subset, say X, of set A such that $x \in X$.

Now, $\quad X \subset A \Rightarrow X \in P(A)$

$\Rightarrow \quad X \in P(B) \quad [\because P(A) = P(B)]$

$\Rightarrow \quad X \subset B$

$\Rightarrow \quad x \in B \quad [\because x \in X \text{ and } X \subset B \Rightarrow x \in B]$

Thus, $x \in A \Rightarrow x \in B$

$\therefore \quad A \subset B. \quad \ldots (1)$

Now, let y be an arbitrary element of B. Then, there exists a subset, say Y, of set B such that $y \in Y$.

Now, $\quad Y \subset B \Rightarrow Y \in P(B)$.