$$(x+y+z)^2\begin{vmatrix}1&0&0\\0&1&-1\\z&x+z&z+x+2y\end{vmatrix}$$

Applying $C_3 \to C_2 + C_3$, we get

$$(x+y+z)^2\begin{vmatrix}1&0&0\\0&1&0\\z&x+z&2(x+y+z)\end{vmatrix}$$

$= 2(x+y+z)^2.\ (x+y+z)$

[Determinant of triangular matrix is product of its diagonal elements]

$= 2(x+y+z)^3$ = R.H.S.

Hence, proved.

12. $\begin{vmatrix}1&x&x^2\\x^2&1&x\\x&x^2&1\end{vmatrix}=(1-x^3)^2.$

Soln.: L.H.S. $=\begin{vmatrix}1&x&x^2\\x^2&1&x\\x&x^2&1\end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$=\begin{vmatrix}1+x+x^2&x&x^2\\1+x+x^2&1&x\\1+x+x^2&x^2&1\end{vmatrix}$$

Taking $(1+x+x^2)$ common from C_1, we get

$$=(1+x+x^2)\begin{vmatrix}1&x&x^2\\1&1&x\\1&x^2&1\end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$, we get

$$=(1+x+x^2)\begin{vmatrix}1-x&x&x^2\\0&1&x\\1-x^2&x^2&1\end{vmatrix}$$

Taking $(1-x)$ common from C_1, we get

$$=(1+x+x^2)(1-x)\begin{vmatrix}1&x&x^2\\0&1&x\\1+x&x^2&1\end{vmatrix}$$

Expanding along C_1, we get

$$=(1+x+x^2)(1-x)\left[1\begin{vmatrix}1&x\\x^2&1\end{vmatrix}+(1+x)\begin{vmatrix}x&x^2\\1&x\end{vmatrix}\right]$$

$=(1-x^3)\,[(1-x^3)+(1+x)\,(x^2-x^2)]=(1-x^3)^2$ = R.H.S.

Hence, proved.

13. $\begin{vmatrix}1+a^2-b^2&2ab&-2b\\2ab&1-a^2+b^2&2a\\2b&-2a&1-a^2-b^2\end{vmatrix}=(1+a^2+\ldots$

Soln.: L.H.S. $=\begin{vmatrix}1+a^2-b^2&2ab&-2b\\2ab&1-a^2+b^2&2a\\2b&-2a&1-a^2-b^2\end{vmatrix}$

Applying $C_1 \to C_1 - bC_3$ and $C_2 \to C_2 + aC_3$, we get

$$=\begin{vmatrix}1+a^2+b^2&0&-2b\\0&1+a^2+b^2&2a\\b(1+a^2+b^2)&-a(1+a^2+b^2)&1-a^2-b^2\end{vmatrix}$$

Taking $(1+a^2+b^2)$ common from C_1 and C_2, we get

$$=(1+a^2+b^2)^2\begin{vmatrix}1&0&-2b\\0&1&2a\\b&-a&1-a^2-b^2\end{vmatrix}$$

Applying $R_3 \to R_3 - bR_1$, we get

$$=(1+a^2+b^2)^2\begin{vmatrix}1&0&-2b\\0&1&2a\\0&-a&1-a^2+b^2\end{vmatrix}$$

Expanding along C_1, we get

$= (1+a^2+b^2)^2\,[1(1-a^2+b^2+2a^2)]$

$= (1+a^2+b^2)^2\,(1+a^2+b^2)$

$= (1+a^2+b^2)^3$ = R.H.S.

Hence, proved.

14. $\begin{vmatrix}a^2+1&ab&ac\\ab&b^2+1&bc\\ca&cb&c^2+1\end{vmatrix}=1+a^2+b^2+c^2.$

Soln.: L.H.S. $=\begin{vmatrix}a^2+1&ab&ac\\ab&b^2+1&bc\\ca&cb&c^2+1\end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\text{L.H.S.}=\begin{vmatrix}1+a(a+b+c)&ab&ac\\1+b(a+b+c)&b^2+1&bc\\1+c(a+b+c)&cb&c^2+1\end{vmatrix}$$

By property 5 and taking $(a+b+c)$ common from C_1 in determinant II, we get

$$\text{L.H.S.}=\begin{vmatrix}1&ab&ac\\1&b^2+1&bc\\1&cb&c^2+1\end{vmatrix}+(a+b+c)\begin{vmatrix}a&ab&ac\\b&b^2+1&bc\\c&cb&c^2+1\end{vmatrix}$$

9. $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$.

Soln.: Let L.H.S. $= \begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}$

Multiplying R_1, R_2 and R_3 by x, y and z respectively, we get

$$\Delta = \frac{1}{xyz}\begin{vmatrix} x^2 & x^3 & xyz \\ y^2 & y^3 & xyz \\ z^2 & z^3 & xyz \end{vmatrix} = \frac{xyz}{xyz}\begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}$$

[Taking xyz common from C

Applying $C_2 \leftrightarrow C_3$, we get

$$-\begin{vmatrix} x^2 & 1 & x^3 \\ y^2 & 1 & y^3 \\ z^2 & 1 & z^3 \end{vmatrix}$$

Applying $C_1 \leftrightarrow C_2$, we get

$$\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} 1 & x^2 & x^3 \\ 0 & y^2 - x^2 & y^3 - x^3 \\ 0 & z^2 - x^2 & z^3 - x^3 \end{vmatrix}$$

Taking $(y-x)$ and $(z-x)$ common from R_2 and R_3 respectively, we get

$$(y-x)(z-x)\begin{vmatrix} 1 & x^2 & x^3 \\ 0 & y+x & y^2+yx+x^2 \\ 0 & z+x & z^2+zx+x^2 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_3$, we get

$$(y-x)(z-x)\begin{vmatrix} 1 & x^2 & x^3 \\ 0 & y-z & y^2-z^2+x(y-z) \\ 0 & z+x & z^2+zx+x^2 \end{vmatrix}$$

Taking $(y-z)$ common from R_2, we get

$$(y-x)(z-x)(y-z)\begin{vmatrix} 1 & x^2 & x^3 \\ 0 & 1 & x+y+z \\ 0 & z+x & z^2+zx+x^2 \end{vmatrix}$$

Expanding along C_1, we get

$= (y-x)(z-x)(y-z)[(z^2+zx+x^2) - (z+x)(x+y+z)]$

$= (y-x)(z-x)(y-z)[z^2+zx+x^2-zx-zy-z^2-x^2-xy-zx]$

$= (y-x)(z-x)(y-z)[-xy-xz-zy]$

$= (x-y)(y-z)(z-x)(xy+yz+zx) =$ R.H.S.

Hence, proved.

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Changing rows into columns, we have

$$\text{L.H.S.} = \begin{vmatrix} 1 & 1 & 1 \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} + (a+b+c)\begin{vmatrix} a & b & c \\ ab & b^2+1 & cb \\ ac & bc & c^2+1 \end{vmatrix}$$

For determinant I we apply, $C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$ and for determinant II we take out a common from C_1, we get

$$\text{L.H.S.} = \begin{vmatrix} 0 & 0 & 1 \\ ab-b^2-1 & b^2+1-cb & bc \\ ac-bc & bc-c^2-1 & c^2+1 \end{vmatrix}$$

$$+a(a+b+c)\begin{vmatrix} 1 & b & c \\ b & b^2+1 & bc \\ c & bc & c^2+1 \end{vmatrix}$$

Applying $C_2 \to C_2 - bC_1$ and $C_3 \to C_3 - cC_1$ in determinant II, we get

$$\begin{vmatrix} 0 & 0 & 1 \\ ab-b^2-1 & b^2+1-cb & bc \\ ac-bc & bc-c^2-1 & c^2+1 \end{vmatrix} + a(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ c & 0 & 1 \end{vmatrix}$$

Expanding along R_1, we have

$\text{L.H.S.} = 1[(ab-b^2-1)(bc-c^2-1)-(ac-bc)(b^2+1-cb)]$
$+a(a+b+c)$

$= [(ab^2c - abc^2 - ab - b^3c + b^2c^2 + b^2 - bc + c^2 + 1$
$-(ab^2c + ac - abc^2 - b^3c - bc + b^2c^2)] + a(a+b+c)$

$= ab^2c - abc^2 - ab - b^3c + b^2c^2 + b^2 - bc + c^2 + 1 - ab^2c - ac$
$+ abc^2 + b^3c + bc - b^2c^2 + a(a+b+c)$

$= -ab + b^2 + c^2 + 1 - ac + a^2 + ab + ac$

$= 1 + a^2 + b^2 + c^2 = \text{R.H.S.}$

Hence, proved.

Choose the correct answer in Exercise 15 and 16.

15. Let A be a square matrix of order 3×3, then $|kA|$ is equal to

(A) $k|A|$ (B) $k^2|A|$

(C) $k^3|A|$ (D) $3k|A|$

Soln.: (C) If A is a square matrix of order n, then $|kA| = k^n|A|$, where k is scalar.

16. Which of the following is correct ?

(A) Determinant is a square matrix.

(B) Determinant is a number associated to a matrix.

(C) Determinant is a number associated to a square matrix.

(D) None of these

Soln.: (C) Determinant is a number associated to a square matrix.

Exercise - 4.3

1. Find area of the ... the point given

Soln.: (i) A

$= \frac{1}{2}\Big[1\Big|\,0$ 3

$= \frac{1}{2}[1(0-$

(ii) Area

$= \frac{1}{2}\Big[2\Big|\,1$ 8

$= \frac{1}{2}[2(1$

$= \frac{1}{2}[-1$

(iii) Ar

$= \frac{1}{2}\Big[-$

$= \frac{1}{2}[-$

$= \frac{1}{2}[-$

$\therefore$ A

2. S collin

Soln.

Appl

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10. (i) $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

(ii) $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

Soln.: (i) L.H.S. = $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix} = (5x+4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, we get

$$(5x+4)\begin{vmatrix} 0 & x-4 & 0 \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix}$$

Expanding along R_1, we get

$$(5x+4)\left[-(x-4)\begin{vmatrix} 1 & 2x \\ 1 & x+4 \end{vmatrix}\right]$$

$= (5x+4)\,[-(x-4)(x+4-2x)]$

$= (5x+4)\,[-(x-4)(-x+4)] = (5x+4)(4-x)(4-x)$

$= (5x+4)(4-x)^2$ = R.H.S.

Hence, proved.

(ii) L.H.S. = $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 3y+k & y & y \\ 3y+k & y+k & y \\ 3y+k & y & y+k \end{vmatrix} = (3y+k)\begin{vmatrix} 1 & y & y \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$, we get

$$(3y+k)\begin{vmatrix} 0 & -k & 0 \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix} = (3y+k)\left[k\begin{vmatrix} 1 & y \\ 1 & y+k \end{vmatrix}\right]$$

$= (3y+k)k[y+k-y] = (3y+k)k^2$ = R.H.S.

Hence, proved.

11. (i) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

(ii) $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$

Soln.: (i) L.H.S. = $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$, we get

$$\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Taking $(a+b+c)$ common from R_1, we get

$$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Applying $C_1 \to C_1 - C_2$, we get

$$(a+b+c)\begin{vmatrix} 0 & 1 & 1 \\ b+c+a & b-c-a & 2b \\ 0 & 2c & c-a-b \end{vmatrix}$$

Taking $(a+b+c)$ common from C_1, we get

$$(a+b+c)^2\begin{vmatrix} 0 & 1 & 1 \\ 1 & b-c-a & 2b \\ 0 & 2c & c-a-b \end{vmatrix}$$

Applying $C_2 \to C_2 - C_3$, we get

$$(a+b+c)^2\begin{vmatrix} 0 & 0 & 1 \\ 1 & -(a+b+c) & 2b \\ 0 & (a+b+c) & c-a-b \end{vmatrix}$$

Taking $(a+b+c)$ common from C_2, we get

$$(a+b+c)^3\begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & 2b \\ 0 & 1 & c-a-b \end{vmatrix}$$

Expanding along R_1, we get

$(a+b+c)^3$ = R.H.S.

Hence, proved.

(ii) L.H.S. = $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} (x+y+z) & -(x+y+z) & 0 \\ 0 & x+y+z & -(x+y+z) \\ z & x & z+x+2y \end{vmatrix}$$

Taking $(x+y+z)$ common from R_1 and R_2, we get

$$(x+y+z)^2\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ z & x & z+x+2y \end{vmatrix}$$

Applying $C_2 \to C_1 + C_2$, we get