and 250, which is a multiple of 4 (*i.e.*, divisible by 4). Also, when we divide 250 by 4, the remainder is 2. Therefore, 250 – 2 = 248 is the largest integer divisible by 4 (*i.e.*, multiple of 4) and lying between 10 and 250. Thus, we have to find the number of terms in an AP with first term = 12, last term = 248, and common difference = 4 (as the number are divisible by 4).

Let there be n terms in the AP Then,

$$a_n = 248 \quad \Rightarrow \quad 12 + (n-1)4 = 248$$

$$\Rightarrow \quad 4(n-1) = 248 - 12 \quad \Rightarrow \quad 4(n-1) = 236$$

$$\Rightarrow \quad n - 1 = \frac{236}{4} = 59 \quad \Rightarrow \quad n = 59 + 1 = 60$$

Hence, there are 60 multiples of 4 between 10 and 250.

15. For what value of n, are the nth terms of the APs : 63, 65, 67, ... and 3, 10, 17, ... are equal ?

Sol. If nth terms of the APs 63, 65, 67, ... and 3, 10, 17, ... are equal. Then,

$$63 + (n-1)2 = 3 + (n-1)7$$

[$\because$ *In 1st AP* $a = 63$, $d = 65 - 63 = 2$ *and in 2nd AP* $a = 3$, $d = 10 - 3 = 7$]

$$\Rightarrow \quad 7(n-1) - 2(n-1) = 63 - 3$$

$$\Rightarrow \quad (n-1)(7-2) = 60$$

$$\Rightarrow \quad 5(n-1) = 60$$

$$\Rightarrow \quad n - 1 = \frac{60}{5} = 12$$

$$\Rightarrow \quad n = 12 + 1 = 13$$

Hence, the 13th terms of the two given APs are equal.

16. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

and $a_n = a_1 + (n-1)d$

$b_n = b_1 + (n-1)d$

$\Rightarrow \quad a_n - b_n = [a_1 + (n-1)d] - [b_1 + (n-1)d]$

$\Rightarrow \quad a_n - b_n = a_1 - b_1$ for all $n \in N$

$\Rightarrow \quad a_{100} - b_{100} = a_1 - b_1 = 100$ [*Given*]

Now, $a_{1000} - b_{1000} = a_1 - b_1$

$\Rightarrow \quad a_{1000} - b_{1000} = 100$ [$\because a_1 - b_1 = 100$]

Hence, the difference between 1000th terms is the same as the difference between 100th terms, *i.e.*, 100.

13. How many three-digit numbers are divisible by 7 ?

Sol. 994 is the last 3-digit number divisible by 7. Thus, we have to determine the number of terms in the list 105, 112, 119, ..., 994.

Clearly, it forms an AP with first term $a = 105$

and common difference $d = 112 - 105 = 7$

Let there be n terms in the AP Then, nth term = 994.

As $a_n = a + (n-1)d$

$\Rightarrow \quad 105 + (n-1)7 = 994$

$\Rightarrow \quad 7(n-1) = 994 - 105$

$\Rightarrow \quad 7(n-1) = 889$

$\Rightarrow \quad n - 1 = \frac{889}{7} = 127$

$\Rightarrow \quad n = 127 + 1 = 128$

Hence, there are 128 numbers of three digit which are divisible by 7.

14. How many multiples of 4 lie between 10 and 250 ?

Sol. We observe that 12 is the first integer between 10

$\Rightarrow (a + 5d) + (a + 9d) = 44$

$\Rightarrow 2a + 14d = 44 \Rightarrow a + 7d = 22$ ...(2)

Subtracting (1) from (2), we get

$2d = 10 \Rightarrow d = 5$

and then from (1),

$a + 25 = 12 \Rightarrow a = -13$

The first three terms are a, $(a + d)$ and $(a + 2d)$

Putting values of a and d, we get -13, $(-13 + 5)$ and $(-13 + 2 \times 5)$

i.e., -13, -8 and -3

19. Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000 ?

Sol. The annual salary drawn by Subba Rao in the years 1995, 1996, 1997, etc. is Rs 5,000, Rs 5,200, Rs 5,400, ..., Rs 7,000.

The list of these nos. is 5000, 5200, 5400, ..., 7000.

It forms an AP $[\because a_2 - a_1 = a_3 - a_2 = 200$

Let $a_n = 7000$

$\Rightarrow 7000 = a + (n - 1)d$

$\Rightarrow 7000 = 5000 + (n - 1)(200)$

$\Rightarrow 200(n - 1) = 7000 - 5000$

$\Rightarrow n - 1 = \frac{2000}{200} = 10$

$\Rightarrow n = 10 + 1 = 11$

Thus, in the 11th year (*i.e.*, in 2005) of his service Subba Rao drew an annual salary of Rs 7,000.

Sol. Let a be the first term and d the common ference.

Here, $a_3 = 16$ and $a_7 - a_5 = 12$

$$\Rightarrow \quad a + 2d = 16 \quad \ldots(1)$$

and, $(a + 6d) - (a + 4d) = 12 \Rightarrow 2d = 12$

$$\Rightarrow \quad d = 6 \quad \ldots(2)$$

Using (2) in (1), we get

$$a + 2 \times 6 = 16$$

$$\Rightarrow \quad a = 16 - 12 = 4$$

Thus, the required AP is $4, 4 + 6, 4 + 2 \times 6, 4 + 3 \times 6,$ ... 4, 10, 16, 22, ...

17. Find the 20th term from the last term of the :3, 8, 13, ..., 253.

Sol. We have, l = Last term = 253

and, d = Common difference = $8 - 3 = 5$

$\therefore$ 20th term from the end $= l - (20 - 1)d$

$$= l - 19d$$

$$= 253 - 19 \times 5$$

$$= 253 - 95 = 158$$

18. The sum of the 4th and 8th terms of an AP is nd the sum of the 6th and 10th terms is 44. Find first three terms of the AP

Sol. Let a be the first term and d the common rence.

Here, $a_4 + a_8 = 24$

$\Rightarrow (a + 3d) + (a + 7d) = 24$

$\Rightarrow \quad 2a + 10d = 24 \quad \ldots(1)$

$\Rightarrow \quad a + 5d = 12$

and, $a_6 + a_{10} = 44$