$\Rightarrow \quad -5k = 10 \quad$ or $\quad 5k = 60$

Exercise - 4.4

Write Minors and Cofactors of the elements of following determinants :

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1. (i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$ (ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$

Soln.: (i) Let $P = \begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

Minor of the element a_{ij} is M_{ij}. Here,

$M_{11} = 3,\ M_{12} = 0,\ M_{21} = -4,\ M_{22} = 2$

For cofactors, we know that $P_{ij} = (-1)^{i+j} M_{ij}$

$\therefore \quad P_{11} = 3, P_{12} = -0 = 0, P_{21} = 4, P_{22} = 3$

(ii) Let $P = \begin{vmatrix} a & c \\ b & d \end{vmatrix}$, Minor of the element a_{ij} is M_{ij}.

Here $M_{11} = d,\ M_{12} = b,\ M_{21} = c,\ M_{22} = a$

For cofactors, we know that $P_{ij} = (-1)^{i+j} M_{ij}$

$\therefore P_{11} = d,\ P_{12} = -b,\ P_{21} = -c,\ P_{22} = a$

2. (i) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ (ii) $\begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

Soln.: (i) Let $P = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$, we have

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$M_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1,\ M_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0,\ M_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$M_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0,\ M_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1,\ M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$M_{31} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0,\ M_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0,\ M_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

For cofactors, we know that $P_{ij} = (-1)^{i+j} M_{ij}$

$P_{11} = 1,\ P_{12} = 0,\ P_{13} = 0,\ P_{21} = 0,\ P_{22} = 1,\ P_{23} = 0,\ P_{31} = 0,$
$P_{32} = 0,\ P_{33} = 1$

(ii) Let $P = \begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$, we have

$M_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 10 + 1 = 11,\ M_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = 6,\ M_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3$

$M_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = -4,\ M_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2,\ M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$M_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = -20,\ M_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -13$

$M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5$

For cofactors, we know that $P_{ij} = (-1)^{i+j} M_{ij}$

$P_{11} = 11,\ P_{12} = -6,\ P_{13} = 3,\ P_{21} = 4,\ P_{22} = 2,\ P_{23} = -1,$
$P_{31} = -20,\ P_{32} = 13,\ P_{33} = 5$

3. Using Cofactors of elements of second row, evaluate

$\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}.$

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Soln.: $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}.$

Cofactors of elements of second row are

$A_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = -(9 - 16) = 7$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 5 & 8 \\ 1 & 3 \end{vmatrix} = 15 - 8 = 7$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -(10 - 3) = -7$

Now, $\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{33}A_{33} = 2 \times 7 + 0 \times 7 + 1 \times (-7)$
$= 14 + 0 - 7 = 7.$

4. Using Cofactors of elements of third column, evaluate

$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}.$

Soln.: Let $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}.$

Cofactors of elements of third column are

$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} = z - y,\ A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = -(z - x)$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = y - x = -(x - y)$

Now,

$\Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$
$\Delta = -yz(y - z) - zx(z - x) - xy(x - y)$
$= zy(z - y) + zx(x - z) + xy(y - x)$
$= yz^2 - y^2z + zx^2 - z^2x + xy^2 - x^2y$
$= x^2z - x^2y + xy^2 - xz^2 + yz^2 - y^2z$
$= x^2(z - y) + x(y^2 - z^2) + yz(z - y)$
$= (z - y)[x^2 - x(y + z) + yz] = (z - y)\{x(x - y) - z(x - y)\}$
$= (z - y)(x - y)(x - z) = (x - y)(y - z)(z - x)$

5. If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ and A_{ij} is Cofactors of a_{ij}, then value of Δ is given by

(A) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$
(B) $a_{11}A_{11} + a_{12}A_{21} + a_{13}A_{31}$
(C) $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$
(D) $a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

Soln. (D) $\Delta = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}.$

Exercise - 4.5

Find adjoint of each of the matrices in Exercises 1 and 2.

1. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Soln.: Let $P = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Let A_{ij} be cofactor of a_{ij} in P. Then, the cofactors of elements of P are given by

$A_{11} = (-1)^{1+1}(4) = 4,$
$A_{12} = (-1)^{1+2}(3) = -3,$
$A_{21} = (-1)^{2+1}(2) = -2$
$A_{22} = (-1)^{2+2}(1) = 1$

$\therefore$ adj $P = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}' = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$