## Sum of the First n Terms of an AP

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### Arithmetic Series

*When we add all the terms of an AP, we get the corresponding arithmetic series.*

If $a_1, a_2, a_3, \ldots, a_n$ are the $n$ terms of an AP, then the corresponding arithmetic series is given by

$$S_n = a_1 + a_2 + a_3 + \ldots\ldots + a_n$$

or

$$S_n = a + (a+d) + (a+2d) + \ldots\ldots + [a+(n-1)d]$$

Here, $S_n$ denotes the sum of first $n$ terms of an AP.

### Sum of the First n Terms of an AP

Consider an AP of $n$ terms with first term $a$ and common difference $d$. Its last term $l$ will be

$$l = a_n = a + (n-1)d$$

Then, the sum of the first $n$ terms of the AP can be written as

$$S_n = a + (a+d) + (a+2d) + \ldots\ldots + (l-2d) + (l-d) + l \qquad \ldots(1)$$

Writing the terms in reverse order, we get

$$S_n = l + (l-d) + (l-2d) + \ldots\ldots + (a+2d) + (a+d) + a \qquad \ldots(2)$$

Adding vertically the corresponding terms of (1) and (2), we get

$$2S_n = \underbrace{(a+l) + (a+l) + (a+l) + \ldots\ldots + (a+l) + (a+l) + (a+l)}_{n \text{ terms}}$$

$$\therefore \quad 2S_n = n(a+l)$$

$$\Rightarrow \quad S_n = \frac{n}{2}(a+l)$$

$$\Rightarrow \quad S_n = [2a + (n-1)d] \qquad [\because l = a_n = a + (n-1)d]$$

Hence, the sum of the first $n$ terms of an AP is given by

$$S_n = \frac{n}{2}[2a + (n-1)d] \qquad \text{or} \qquad S_n = \frac{n}{2}(a+l)$$

By knowing any three of the four quantities $S_n$, $a$, $d$ and $n$ ; we can find the fourth quantity by using the first formula.

The second formula for $S_n$ is useful when the first and the last terms of an AP are given but the common difference $d$ is not given.

### To find nth Term from the Sum of n Terms of an AP

The nth term of an AP can be found by subtracting the sum of first $(n-1)$ terms of the AP from the sum of first $n$ terms of the same AP. That is,

$$a_n = S_n - S_{n-1}$$

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## Solved Examples

EX - 5.3

**Example** Find the sum of following APs : [NCERT]

Q1 (*i*) 2, 7, 12,..., to 10 terms

(*ii*) −37, −33, −29, ..., to 12 terms

(*iii*) 0.6, 1.7, 2.8,..., to 100 terms

(*iv*) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$, to 11 terms

**Solution.** (*i*) 2, 7, 12, ..., to 10 terms. Here, $a = 2$, $d = 7 - 2 = 5$, $n = 10$, $S_{10} = ?$

$$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad S_{10} = \frac{10}{2}[2 \times 2 + (10-1)5] = 5[4+45] = 5 \times 49 = \mathbf{245}.$$

Q19 Example 19 Suba Rao started work in 1995 at an annual salary of ₹5000 and received ₹200 raise each year. In what year did his income reach ₹7000 ? [NCERT]

**Solution.** Suba Rao's salary in the years 1995, 1996, 1997,... respectively is

₹5000, ₹5200, ₹5400,...

X It is an AP with $a = 5000$ and $d = 200$ Ex-5.2

Suppose Suba Rao's salary reaches ₹7000 in the $n$th year. Then,

$$a_n = 7000$$

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$$\Rightarrow \quad a+(n-1)d = 7000 \quad \Rightarrow \quad 5000+(n-1)\times 200 = 7000$$

$$\Rightarrow \quad (n-1)\times 200 = 2000$$

$$\Rightarrow \quad n-1 = 10 \quad \Rightarrow \quad n = 11$$

Hence, Suba Rao's salary reaches ₹7000 in the 11th year *i.e.*, **in 2005**

Q20 Example 20 Ramkalis saved ₹5 in the first week of a year and then increased her weekly savings by ₹1.75. If in the $n$th week, her weekly savings become ₹20.75, find $n$.

**Solution.** Ramkali's savings in the consecutive weeks are :

₹5, ₹$(5+1.75)$, ₹$(5+2\times 1.75)$, ₹$(5+3\times 1.75)$,...

Clearly, it is an AP with $a = 5$ and $d = 1.75$

Given, $$a_n = 20.75$$

$$\Rightarrow \quad a+(n-1)d = 20.75 \quad \Rightarrow \quad 5+(n-1)\times 1.75 = 20.75$$

$$\Rightarrow \quad (n-1)\times 1.75 = 15.75$$

$$\Rightarrow \quad n-1 = \frac{15.75}{1.75} = 9 \quad \Rightarrow \quad n = 10$$

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Q1

(ii) $-37, -33, -29, \ldots$, to 12 terms

Here, $a = -37$, $d = -33-(-37) = -33+37 = 4$, $n = 12$, $S_{12} = ?$

$\because \quad S_n = \frac{n}{2}[2a+(n-1)d]$

$\therefore \quad S_{12} = \frac{12}{2}[2\times(-37)+(12-1)\times(4)] = 6[-74+44] = -6\times 30 = -\mathbf{180}.$

(iii) $0.6, 1.7, 2.8, \ldots$, to 100 terms

Here, $a = 0.6$, $d = 1.7-0.6 = 1.1$, $n = 100$, $S_{100} = ?$

$\because \quad S_n = \frac{n}{2}[2a+(n-1)d]$

$\therefore \quad S_{100} = \frac{100}{2}[2\times(0.6)+(100-1)\times 1.1] = 50[1.2+108.9] = 50\times 110.1 = 5505.0 = \mathbf{5505}.$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \ldots$, to 11 terms

Here, $a = \frac{1}{15}$, $d = \frac{1}{12} - \frac{1}{15} = \frac{15-12}{12\times 15} = \frac{3}{12\times 15} = \frac{1}{60}$, $n = 11$, $S_{11} = ?$

$\because \quad S_n = \frac{n}{2}[2a+(n-1)d]$

$\therefore \quad S_{11} = \frac{11}{2}\left[2\times\frac{1}{15}+(11-1)\times\frac{1}{60}\right] = \frac{11}{2}\left[\frac{2}{15}+\frac{10}{60}\right] = \frac{11}{2}\left[\frac{8+10}{60}\right] = \frac{11}{2}\times\frac{18}{60} = \mathbf{\frac{33}{20}}.$

[NCERT]

Q2 **Example 6** Find the sums given below :

(i) $7+10\frac{1}{2}+14+\ldots+84$ (ii) $34+32+30+\ldots+10$ (iii) $-5+(-8)+(-11)+\ldots+(-230)$

**Solution.** (i) Let $S_n = 7+10\frac{1}{2}+14+\ldots+84$

Here, $a = 7$, $d = \frac{21}{2} - 7 = \frac{7}{2}$, $a_n = 84 = l$

$\therefore \quad a_n = 84$

$\Rightarrow \quad a+(n-1)d = 84 \quad$ or $\quad 7+(n-1)\times\frac{7}{2} = 84$

or $\quad (n-1)\times\frac{7}{2} = 84-7 = 77 \quad$ or $\quad n-1 = 77\times\frac{2}{7} = 22$

$\Rightarrow \quad n = 23$

$\because \quad S_n = \frac{n}{2}[a+l]$

$\therefore \quad S_{23} = \frac{23}{2}[7+84] = \frac{23}{2}\times 91 = \frac{2093}{2} = \mathbf{1046.5}.$

(ii) Let $S_n = 34+32+30+\ldots+10$

Here, $a = 34$, $d = 32-34 = -2$, $a_n = 10 = l$

$\because \quad a_n = 10 \quad \Rightarrow \quad a+(n-1)d = 10$

$34+(n-1)(-2) = 10 \quad \Rightarrow \quad -2(n-1) = 10-34 = -24$

$n-1 = 12$ or $n = 13$

$\therefore \quad S_{13} = \frac{13}{2}[a+l] = \frac{13}{2}[34+10] = \frac{13}{2}\times 44 = 13\times 22 = \mathbf{286}.$