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Q2

Soln.: Let $P = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$. Let A_{ij} be cofactors of a_{ij} in A. Then,

$$A_{11} = (-1)^{1+1}\begin{bmatrix} 3 & 5 \\ 0 & 1 \end{bmatrix} = 3 - 0 = 3$$

$$A_{12} = (-1)^{1+2}\begin{bmatrix} 2 & 5 \\ -2 & 1 \end{bmatrix} = -(2+10) = -12$$

$$A_{13} = (-1)^{1+3}\begin{bmatrix} 2 & 3 \\ -2 & 0 \end{bmatrix} = 0 + 6 = 6$$

$$A_{21} = (-1)^{2+1}\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} = -(-1-0) = 1$$

$$A_{22} = (-1)^{2+2}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} = 1 + 4 = 5$$

$$A_{23} = (-1)^{2+3}\begin{bmatrix} 1 & -1 \\ -2 & 0 \end{bmatrix} = -(0-2) = 2$$

$$A_{31} = (-1)^{3+1}\begin{bmatrix} -1 & 2 \\ 3 & 5 \end{bmatrix} = -5 - 6 = -11$$

$$A_{32} = (-1)^{3+2}\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} = -(5-4) = -1$$

$$A_{33} = (-1)^{3+3}\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = 3 + 2 = 5$$

$$\therefore \quad \text{adj } P = \begin{bmatrix} 3 & -12 & 6 \\ 1 & 5 & 2 \\ -11 & -1 & 5 \end{bmatrix}' = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$$

Verify : A (adj A) = (adj A) A = $|A|\,I$ in Exercises 3 and 4.

3. $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

Soln.: Let $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

$|A| = -12 + 12 \Rightarrow |A| = 0$

Let A_{ij} be co-factors of a_{ij} in A. Then, the co-factors of elements of A are given by

$A_{11} = (-1)^{1+1}(-6) = -6,\ A_{12} = (-1)^{1+2}(-4) = 4$

$A_{21} = (-1)^{2+1}(3) = -3,\ A_{22} = (-1)^{2+2}(2) = 2$

$$\Rightarrow \quad \text{adj } A = \begin{bmatrix} -6 & 4 \\ -3 & 2 \end{bmatrix}' = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$\therefore \quad A(\text{adj } A) = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}\begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$(\text{adj } A)\, A = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ and}$$

$|A|\,I = O$ as $|A| = 0$

Hence, A (adj A) = (adj A) A = $|A|\,I$

4. $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

Soln.: Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

Let A_{ij} be cofactor of a_{ij} in A. Then, the cofactors of elements of A are given by

$$A_{11} = (-1)^{1+1}\begin{bmatrix} 0 & -2 \\ 0 & 3 \end{bmatrix} = 0$$

$$A_{12} = (-1)^{1+2}\begin{bmatrix} 3 & -2 \\ 1 & 3 \end{bmatrix} = -(9+2) = -11$$

$$A_{13} = (-1)^{1+3}\begin{bmatrix} 3 & 0 \\ 1 & 0 \end{bmatrix} = 0$$

$$A_{21} = (-1)^{2+1}\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix} = -(-3-0) = 3$$

$$A_{22} = (-1)^{2+2}\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = 3 - 2 = 1$$

$$A_{23} = (-1)^{2+3}\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} = -(0+1) = -1$$

$$A_{31} = (-1)^{3+1}\begin{bmatrix} -1 & 2 \\ 0 & -2 \end{bmatrix} = 2$$

$$A_{32} = (-1)^{3+2}\begin{bmatrix} 1 & 2 \\ 3 & -2 \end{bmatrix} = -(-2-6) = 8$$

$$A_{33} = (-1)^{3+3}\begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix} = 0 + 3 = 3$$

$$\therefore \text{ adj } A = \begin{bmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{bmatrix}' = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$\text{Now, } A(\text{adj } A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0+0+0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} = 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 11I$$

$$(\text{Adj } A)\, A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

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$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} = 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 11I$$

Also, $|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix} = 1(0) + 1(9+2) + 2(0) = 11$

Hence, $A\,(\text{Adj } A) = (\text{Adj } A)\,A = 11I = |A|\,I$.

Find the inverse of each of the matrices (if it exists) given in Exercises 5 to 11.

5. $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

Soln.: Let $A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix} = 6 + 8 = 14 \neq 0.$

So, A is a non-singular matrix and therefore, it is invertible. Let A_{ij} be cofactor of a_{ij} in A. Then, the cofactors of elements of A are given by

$A_{11} = (-1)^{1+1}(3) = 3,\ A_{12} = (-1)^{1+2}(4) = -4,$

$A_{21} = (-1)^{2+1}(-2) = 2,\ A_{22} = (-1)^{2+2}(2) = 2$

$\therefore\quad \text{adj } A = \begin{bmatrix} 3 & -4 \\ 2 & 2 \end{bmatrix}' = \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$

Hence, $A^{-1} = \dfrac{1}{|A|}\,\text{adj } A = \dfrac{1}{14}\begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$

6. $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

Soln.: Let $A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} -1 & 5 \\ -3 & 2 \end{vmatrix} = -2 + 15 = 13 \neq 0.$

So, A is a non-singular matrix and therefore, it is invertible. Let A_{ij} be cofactor of a_{ij} in A. Then, the cofactors of elements of A are given by

$A_{11} = (-1)^{1+1}(2) = 2,\ A_{12} = (-1)^{1+2}(-3) = 3$

$A_{21} = (-1)^{2+1}(5) = -5,\ A_{22} = (-1)^{2+2}(-1) = -1$

$\therefore\quad \text{adj } A = \begin{bmatrix} 2 & 3 \\ -5 & -1 \end{bmatrix}' = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

Hence, $A^{-1} = \dfrac{1}{|A|}(\text{adj } A) = \dfrac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Soln.: Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Then, $|A| = 10 \neq 0.$

So, A is a non-singular matrix and therefore, it is invertible. Let A_{ij} be cofactors of a_{ij} in A. Then,

$A_{11} = (-1)^{1+1}\begin{bmatrix} 2 & 4 \\ 0 & 5 \end{bmatrix} = 10,\ A_{12} = (-1)^{1+2}\begin{bmatrix} 0 & 4 \\ 0 & 5 \end{bmatrix} = 0$

$A_{13} = (-1)^{1+3}\begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} = 0,\ A_{21} = (-1)^{2+1}\begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} = -10$

$A_{22} = (-1)^{2+2}\begin{bmatrix} 1 & 3 \\ 0 & 5 \end{bmatrix} = 5,\ A_{23} = (-1)^{2+3}\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = 0$

$A_{31} = (-1)^{3+1}\begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix} = 2,\ A_{32} = (-1)^{3+2}\begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix} = -4$

$A_{33} = (-1)^{3+3}\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} = 2$

$\therefore\quad \text{adj } A = \begin{bmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{bmatrix}' = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$

Hence, $A^{-1} = \dfrac{1}{|A|}(\text{adj } A) = \dfrac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

Soln.: Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

Then, $|A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix} = 1\begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} = -3 - 0 = -3 \neq 0$

So, A is non-singular matrix and therefore, it is invertible. Let A_{ij} be cofactors of a_{ij} in A. Then, the cofactor of elements of A are given by

$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} = (-3 - 0) = -3$

$A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix} = -(-3 - 0) = 3$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix} = 6 - 15 = -9$

$A_{21} = (-1)^{2+1}\begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix} = 0$

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Sol. (*i*) Let $f(x) = y = 2 - 3x$, *i.e.*, $x = \dfrac{2 - y}{3}$.

Now, $x > 0$, *i.e.*, $2 - y > 0$ or $y < 2$.

$\therefore$ Range $(f) = \{y : y \in R$ and $y < 2\}$.

(*ii*) Let $f(x) = y = x^2 + 2$, *i.e.*, $x^2 = y - 2$ or $x = \sqrt{y - 2} \Rightarrow y$

$\therefore$ Range $(f) = \{y : y \in R$ and $y \geq 2\}$.

(*iii*) $f(x) = x$, x is a real number

$x = f(x) = y =$ a real number

$\therefore$ Range $(f) = \{y : y \in R\}$.