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Q2 (iii) Let $S_n = -5 + (-8) + (-11) + \ldots + (-230)$

Here $a = -5$, $d = -8 - (-5) = -3$, $a_n = -230 = l$

$\because \quad a_n = -230$

$\Rightarrow \quad a + (n-1)d = -230 \qquad \Rightarrow -5 + (n-1)(-3) = -230$

$-3(n-1) = -230 + 5 = -225$

$n - 1 = \frac{225}{3} = 75$

$\Rightarrow \quad n = 75 + 1 = 76$

$\therefore \quad S_{76} = \frac{76}{2}[a + l] = 38[-5 + (-230)] = -38 \times 235 = -\mathbf{8930}.$

Q3 Example 6

(i) Given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n

(ii) Given $a = 7$, $a_{13} = 35$, find d and S_{13}

(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12}

(iv) Given $a_3 = 15$, $S_{10} = 120$, find d and a_{10}

(v) Given $d = 5$, $S_9 = 75$, find a and a_9

(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n

(vii) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d

(viii) Given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d

(x) Given $l = 28$, $S = 144$ and there are total 9 terms. Find a.

Solution. (i) Given : $a = 5$, $d = 3$, $a_n = 50$, $n = ?$, $S_n = ?$

$\because \quad a_n = 50 \Rightarrow a + (n-1)d = 50$

$5 + (n-1) \times 3 = 50 \Rightarrow 3(n-1) = 45$

$n - 1 = \frac{45}{3} = 15$ or $n = \mathbf{16}$

Now, $\quad S_n = \frac{n}{2}[a + a_n] \qquad \Rightarrow S_{16} = \frac{16}{2}[5 + 50] = 8 \times 55 = \mathbf{440}.$

(ii) Given : $a = 7$, $a_{13} = 35$, $d = ?$, $S_{13} = ?$

$\because \quad a_{13} = 35 \qquad \Rightarrow a + 12d = 35$

$\Rightarrow \quad 7 + 12d = 35 \qquad \Rightarrow 12d = 35 - 7 = 28$

$\Rightarrow \quad d = \frac{28}{12} = \frac{7}{3}$

$\therefore \quad S_{13} = \frac{13}{2}[a + a_{13}] = \frac{13}{2}[7 + 35] = \frac{13}{2} \times 42 = 13 \times 21 = \mathbf{273}.$

(iii) Given : $a_{12} = 37$, $d = 3$, $a = ?$, $S_{12} = ?$

$\because \quad a_{12} = 37$

$\Rightarrow \quad a + 11d = 37$

or $\quad a + 11 \times 3 = 37$ or $a = 4$

$\therefore \quad S_{12} = \frac{12}{2}[a + a_{12}] = 6[4 + 37] = 6 \times 41 = \mathbf{246}.$

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(*iv*) Given : $a_3 = 15$, $S_{10} = 120$, $d = ?$, $a_{10} = ?$

$$a_3 = 15 \quad \Rightarrow a + 2d = 15 \text{ or } a = 15 - 2d$$

Also, $$S_{10} = \frac{10}{2}[2a + (n-1)d]$$ $\left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$

$$120 = 5[2 \times (15-2d) + (10-1)d]$$

$$24 = 30 - 4d + 9d$$

$\therefore$ $$5d = 24 - 30 = -6 \quad \text{or } d = -\frac{6}{5}$$

$\Rightarrow$ $$a = 15 - 2 \times \left(-\frac{6}{5}\right) = \frac{87}{5}$$

$\therefore$ $$a_{10} = a + 9d = \frac{87}{5} + 9 \times \left(-\frac{6}{5}\right) = \frac{87-54}{5} = \frac{33}{5} = 6.6.$$

(*v*) Given : $d = 5$, $S_9 = 75$, $a = ?$, $a_9 = ?$

$$S_9 = \frac{9}{2}[2a + (9-1)d]$$ $\left[\because S_n = \frac{n}{2}[2a+(n-1)d]\right]$

$$75 = \frac{9}{2}[2a + 8 \times 5]$$

$$75 = 9a + \frac{9}{2} \times 8 \times 5 \quad \text{or} \quad 75 = 9a + 180$$

$\Rightarrow$ $$9a = 75 - 180 = -105 \quad \text{or} \quad a = -\frac{105}{9} = -\frac{35}{3}$$

$\therefore$ $$a_9 = a + 8d = -\frac{35}{3} + 8 \times 5 = \frac{-35 + 40 \times 3}{3} = \frac{85}{3}$$

Hence, $a = -\frac{35}{3}$ and $a_9 = \frac{85}{3}$.

(*vi*) Given : $a = 2$, $d = 8$, $S_n = 90$, $n = ?$, $a_n = ?$

$\because$ $$S_n = \frac{n}{2}[2a + (n-1)d] \quad \Rightarrow 90 = \frac{n}{2}[2 \times 2 + (n-1) \times 8]$$

$$180 = 4n + 8n^2 - 8n \quad \text{or} \quad 8n^2 - 4n - 180 = 0$$

or $$2n^2 - n - 45 = 0 \quad \text{or} \quad 2n^2 - 10n + 9n - 45 = 0$$

or $$2n(n-5) + 9(n-5) = 0 \quad \text{or} \quad (n-5)(2n+9) = 0$$

$\Rightarrow n = 5$ or $n = -\frac{9}{2}$

$n = -\frac{9}{2}$ is not possible, so $n = 5$

$\therefore$ $$a_5 = 2 + (5-1) \times 8 = 2 + 32 = 37.$$

(*vii*) Given : $a = 8$, $a_n = 62$, $S_n = 210$, $n = ?$, $d = ?$

$$S_n = \frac{n}{2}[a + a_n]$$

$$210 = \frac{n}{2}[8 + 62] = \frac{n}{2} \times 70 = 35n$$

$\Rightarrow$ $$n = \frac{210}{35} = 6$$

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(n-1)d

Also, $a_n = 62 \Rightarrow a_6 = 62$

$\Rightarrow a+5d = 62$ or $5d = 62 - a = 62 - 8 = 54$

$\therefore d = \frac{54}{5} = 10.8.$

(viii) Given : $a_n = 4, d = 2, S_n = -14, n = ?, d = ?$

$$S_n = \frac{n}{2}[a+a_n] \Rightarrow -14 = \frac{n}{2}[a+4]$$

$$-28 = na + 4n \Rightarrow a = \frac{-4n-28}{n}$$

Also, $a_n = 4 \Rightarrow a+(n-1)d = 4$

$$a+(n-1)2 = 4 \Rightarrow \frac{-4n-28}{n}+(n-1)2 = 4$$

(n-1)d

$$-4n-28+2n^2-2n = 4n \Rightarrow 2n^2-10n-28 = 0$$

or $n^2-5n-14 = 0$ or $n^2-7n+2n-14 = 0$

or $n(n-7)+2(n-7) = 0$ or $(n-7)(n+2) = 0$

$\Rightarrow n = 7$ or $n = -2$

But $n = -2$ is not possible, so $n = 7$.

$$\therefore a = \frac{-4\times 7-28}{7} = -\frac{56}{7} = -8.$$

(ix) Given : $a = 3, n = 8, S = 192, d = ?$

$S_8 = 192$ [Given]

$$\frac{n}{2}[2a+(n-1)d] = 192 \Rightarrow \frac{8}{2}[2\times 3+(8-1)d] = 192$$

$24+28d = 192$ or $28d = 192-24 = 168$

$$\Rightarrow d = \frac{168}{28} = \frac{24}{4} = 6.$$

(x) Given : $l = 28, S = 144, n = 9, a = ?$

$l = 28$

$$S_n = \frac{n}{2}[a+l] \Rightarrow 144 = \frac{9}{2}[a+28]$$

$\frac{2}{9}\times 144 = a+28$ or $32 = a+28$

$\Rightarrow a = 32-28 = 4.$

94 **Example 60** How many terms of the AP : 9, 17, 25, ... must be taken to give a sum of 636 ? [NCERT]

Solution. Given the AP : 9, 17, 25,.... Here, $a = 9, d = 17-9 = 8$

Let $S_n = 636$

Then, $\frac{n}{2}[2a+(n-1)d] = 636 \Rightarrow \frac{n}{2}[2\times 9+(n-1)8] = 636$

$\Rightarrow 9n+n(n-1)\times 4 = 636 \Rightarrow 4n^2-4n+9n = 636 = 0$

$\Rightarrow 4n^2+5n-636 = 0 \Rightarrow 4n^2+53n-48n-636 = 0$

$\Rightarrow n(4n+53)-12(4n+53) = 0 \Rightarrow (4n+53)(n-12) = 0$

$n-12 = 0$ | $n = \frac{-53}{12}$ not

$n = 12$

Ans