XI 27/4

(*ii*) Let $f(x) = y = \sqrt{x-1}$

$\therefore$ y is well defined for all values of $x \geq 1$.

$\therefore$ Range : $\{y : y \in R, y \geq 0\} = [0, \infty)$.

5. Find the domain and range of the real function f defined by $f(x) = |x - 1|$.

Sol. (*i*) $f(x) = |x - 1|$ is defined for all real values of x.

$\therefore$ Domain of $f = \{x : x \in R\} = R$.

(*ii*) $f(x) = |x - 1|$ can acquire only non-negative values.

$\therefore$ Range $= \{y : y \geq 0\} = R^+$.

6. Let $f = \left\{\left(x, \dfrac{x^2}{1+x^2}\right)\right\}$ be a function from R into R. Determine the range of f.

Sol. Let $y = f(x) = \dfrac{x^2}{1+x^2}$.

$f(x)$ is positive for all values of x.

When $x = 0, y = 1$. Also, denominator > numerator.

$\therefore$ Range of $f = \{y : y \in R \text{ and } y \in (0, 1)\}$

7. Let $f, g : R \to R$ be defined respectively by $f(x) = x + 1$, $g(x) = 2x - 3$. Find $f + g$, $f - g$ and $\dfrac{f}{g}$.

Sol. f, g are defined for all $x \in R$.

(*i*) $(f + g)(x) = f(x) + g(x) = x + 1 + 2x - 3 = 3x - 2$.

(*ii*) $(f - g)(x) = f(x) - g(x) = (x + 1) - (2x - 3) = -x + 4$.

(*iii*) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x+1}{2x-3}$, $x \in R$ and $x \neq \dfrac{3}{2}$.

8. Let $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ be a function from Z into Z defined by $f(x) = ax + b$ for some integers a and b. Determine a and b.

Sol. We have : $f(x) = ax + b$.

For $x = 1, f(x) = 1$.

$\therefore$ $1 = a \times 1 + b$ or $a + b = 1$ ...(1)

For $x = 2, f(x) = 3$.

XI 27/4

Q1 The relation g is defined by $g(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$.

Show that f is a function and g is not a function.

25|

Sol. (i) $f(x) = x^2$ is well defined in the interval $0 \le x \le 3$

Also, $f(x) = 3x$ is well defined in the interval $3 \le x \le 10$

At $x = 3$ from $f(x) = x^2, f(3) = 3^2 = 9$

from $g(x) = 3x, g(3) = 3 \times 3 = 9$.

$\therefore f$ is defined at $x = 3$. Hence f is a function.

(ii) $g(x) = x^2$ is well defined in the interval $0 \le x \le 2$.

$g(x) = 3x$ is also well defined in the interval $2 \le x \le 10$

But at $x = 2$, $g(x) = x^2 = 2^2 = 4$.

Also, $g(x) = 3x = 3 \times 2 = 6$.

At $x = 2$, relation on g has two values.

$\therefore$ Relation g is not a function.

2. $f(x) = x^2$, find $\dfrac{f(1.1) - f(1)}{1.1 - 1}$.

Sol. $f(x) = x^2$ is $f(1.1) = 1.1^2 = 1.21$.

$f(1) = 1^2 = 1$.

$$\therefore \frac{f(1.1) - f(1)}{1.1 - 1} = \frac{1.21 - 1}{1.1 - 1} = \frac{0.21}{0.1} = 2.1.$$

3. **Find the domain of the function** $f(x) = \dfrac{x^2 + 2x + 1}{x^2 - 8x + 12}$.

Sol. $$f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12} = \frac{(x+1)^2}{(x-2)(x-6)}.$$

The function f is not defined at $x = 2, 6$.

$\therefore$ Domain of $f = \{x : x \in R \text{ and } x \ne 2, x \ne 6\} = \{x : x \in R - \{2, 6\}\}$

4. **Find the domain and range of real function f defined by** $f(x) = \sqrt{x - 1}$.

Sol. (i) $f(x) = \sqrt{x-1}$, f is not defined for $x - 1 < 0$ or $x < 1$

Domain $(f) = \{x : x \ge 1\} = [1, \infty)$.