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$$=\begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}=\begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Now, $A^3-6A^2+9A-4I$

$$=\begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}-6\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}+9\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}-4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 22-36+18-4 & -21+30-9+0 & 21-30+9+0 \\ -21+30-9-0 & 22-36+18-4 & -21+30-9+0 \\ 21-30+9+0 & -21+30-9+0 & 22-36+18-4 \end{bmatrix}$$

$$=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}=O$$

Hence, $A^3-6A^2+9A-4I=O$

Now, $A^3-6A^2+9A-4I=O \Rightarrow 4I=A^3-6A^2+9A$

Multiplying both sides by A^{-1}, we get

$$\Rightarrow A^{-1}=\frac{1}{4}A^2-\frac{6}{4}A+\frac{9}{4}I$$

$$=\frac{1}{4}\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}-\frac{6}{4}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}+\frac{9}{4}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$=\frac{1}{4}\begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix}=\frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

17. Let A be a non-singular square matrix of order 3×3. Then, $|\operatorname{adj} A|$ is equal to

(A) $|A|$ (B) $|A|^2$

(C) $|A|^3$ (D) $3|A|$

Soln.: (B) : For any $n \times n$ matrix A, $\det(\operatorname{adj} A) = |A|^{n-1}$

(It holds for singular and non-singular matrices.)

18. If A is an invertible matrix of order 2, then $\det(A^{-1})$ is equal to

(A) $\det(A)$ (B) $\dfrac{1}{\det(A)}$

(C) 1 (D) 0

Soln.: (B) When A is an invertible matrix of order 2,

$AA^{-1}=I_2=A^{-1}A$, where I_2 is identity matrix of order 2.

$\Rightarrow \det(AA^{-1})=\det I \Rightarrow \det A.\det(A^{-1})=1$

$\Rightarrow \det(A^{-1})=\dfrac{1}{\det A}$, $\det A \neq 0$

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$$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a+b & 2a \\ a & a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow 4 + a = 0 \Rightarrow a = -4$

Also, $3 + a + b = 0 \Rightarrow b = -3 + 4 \Rightarrow b = 1$

Hence, $a = -4$, $b = 1$

15. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$,

show that $A^3 - 6A^2 + 5A + 11I = O$. Hence, find A^{-1}.

Soln.: We have

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \therefore A^2 = AA = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1\cdot1+1\cdot1+1\cdot2 & 1\cdot1+1\cdot2+1\cdot(-1) & 1\cdot1+1\cdot(-3)+1\cdot3 \\ 1\cdot1+2\cdot1+(-3)\cdot2 & 1\cdot1+2\cdot2+(-3)\cdot(-1) & 1\cdot1+2\cdot(-3)+(-3)\cdot(3) \\ 2\cdot1+(-1)\cdot1+3\cdot2 & 2\cdot1+(-1)\cdot2+3\cdot(-1) & 2\cdot1+(-1)\cdot(-3)+3\cdot3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

and, $A^3 = A^2A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 4\cdot1+2\cdot1+1\cdot2 & 4\cdot1+2\cdot2+1\cdot(-1) & 4.1+2\cdot(-3)+1\cdot3 \\ -3\cdot1+8\cdot1+(-14)\cdot2 & -3\cdot1+8\cdot2+(-14)\cdot(-1) & -3\cdot1+8\cdot(-3)+(-14)\cdot3 \\ 7\cdot1+(-3)\cdot1+14\cdot2 & 7\cdot1+(-3)\cdot2+14\cdot(-1) & 7\cdot1+(-3)\cdot(-3)+14\cdot3 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

Now, L.H.S. $= A^3 - 6A^2 + 5A + 11I$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} + \begin{bmatrix} -24 & -12 & -6 \\ 18 & -48 & 84 \\ -42 & 18 & -84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 8-24+5+11 & 7-12+5+0 & 1-6+5+0 \\ -23+18+5+0 & 27-48+10+11 & -69+84-15+0 \\ 32-42+10+0 & -13+18-5+0 & 58-84+15+11 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O \Rightarrow A^3 - 6A^2 + 5A + 11I = O$$

Hence, proved.

Now, $A^3 - 6A^2 + 5A + 11I = O$

$\Rightarrow 11I = -A^3 + 6A^2 - 5A$

Multiplying (i) by A^{-1}, we get

$11A^{-1}I = -A^{-1}A^3 + 6A^{-1}A^2 - 5A^{-1}A \Rightarrow 11A^{-1} = -A^2 + 6A - 5$

$$\Rightarrow A^{-1} = -\frac{1}{11}A^2 + \frac{6}{11}A - \frac{5}{11}I$$

$$= -\frac{1}{11}\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + \frac{6}{11}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} - \frac{5}{11}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} -4+6-5 & -2+6+0 & -1+6+0 \\ 3+6+0 & -8+12-5 & 14-18+0 \\ -7+12+0 & 3-6+0 & -14+18-5 \end{bmatrix}$$

$$= \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

16. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$,

verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence, find A^{-1}.

Soln.: We have

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \therefore A^2 = AA = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

and $A^3 = A^2A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$