..., 500

20. Example Show that $a_1, a_2, a_3, \ldots, a_n$ form an AP where a_n is defined as below :

(i) $a_n = 3 + 4n$ (ii) $a_n = a - 5n$

750.

Also, find the sum of the first fifteen terms in each case. [CBSE D O4C]

Solution. (i) Given : $a_n = 3 + 4n$. Putting $n = 1, 2, 3, 4, \ldots$, we get

$a_1 = 3 + 4 \times 1 = 7, \quad a_2 = 3 + 4 \times 2 = 11, \quad a_3 = 3 + 4 \times 3 = 15, \quad a_4 = 3 + 4 \times 4 = 19$, and so on.

Ex- 5.3

The list of numbers is 7, 11, 15, 19,...

Here, $11 - 7 = 15 - 11 = 19 - 15 = 4$, and so on.

Hence, the above list of numbers forms an AP with $a = 7$, and $d = 4$

Also, $S_{15} = \frac{n}{2}[2a + (n-1)d]$, (where $n = 15$)

$$= \frac{15}{2}[14 + 14 \times 4] = \frac{15}{2} \times 70 = 525.$$

is

(ii) Given : $a_n = 9 - 5n$. Putting $n = 1, 2, 3, 4, \ldots$, we get

$a_1 = 9 - 5 \times 1 = 4, \quad a_2 = 9 - 5 \times 2 = -1, \quad a_3 = 9 - 5 \times 3 = -6, \quad a_4 = 9 - 5 \times 4 = -11$, and so on.

The list of numbers is 4, −1, −6, −11,...

Here, $-1 - 4 = -6 - (-1) = -11 - (-6) = -5$

Hence, the above list of numbers is an AP with $a = 4$ and $d = -5$

Also, $S_{15} = \frac{n}{2}[2a + (n-1)d]$, (where $n = 15$)

D 07C]

$$= \frac{15}{2}[8 + 14 \times (-5)] = \frac{15}{2} \times (-62) = 15 \times (-31) = -465.$$

B. ~~Example 85~~ The sum of n terms of an AP is $3n^2 + 5n$. Find the AP. Hence, find its 16th term. [CBSE D 08]

Solution. Here, $S_n = 3n^2 + 5n$

$\Rightarrow \quad S_{n-1} = 3(n-1)^2 + 5(n-1) = 3(n^2 + 1 - 2n) + 5n - 5 = 3n^2 - n - 2$

The nth term of the AP will be

$$a_n = S_n - S_{n-1} = 3n^2 + 5n - 3n^2 + n + 2 = 6n + 2$$

$\Rightarrow \quad a_1 = 6 \times 1 + 2 = 8, \; a_2 = 6 \times 2 + 2 = 14, \; a_3 = 6 \times 3 + 2 = 20, \ldots$

Hence, the AP is 8, 14, 20,... . Also, $a_{16} = 6 \times 16 + 2 = 98$.

B ~~Example 86~~ If S_n, the sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$, then find its nth term. [CBSE D 09]

2 as

Solution. Here, $S_n = 3n^2 - 4n$

$\Rightarrow \quad S_{n-1} = 3(n-1)^2 - 4(n-1) = 3(n^2 - 2n + 1) - 4n + 4 = 3n^2 - 10n + 7$

The nth term of the AP will be

$$a_n = S_n - S_{n-1} = (3n^2 - 4n) - (3n^2 - 10n + 7) = 6n - 7.$$

Q11 Example If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that S_1) ? What is the sum of first two terms ? What is the second term ? Similarly, find the 3rd, the 10th and the nth terms.

Solution. Here, $S_n = 4n - n^2$

First term, $a_1 = S_1 = 4 \times 1 - 1^2 = 3$

Sum of first two terms, $S_2 = 4 \times 2 - 2^2 = 4$

Second term, $a_2 = S_2 - S_1 = 4 - 3 = 1$

0.

X (5.3)

Q11

$S_3 = 4 \times 3 - 3^2 = 3$ $\therefore$ $a_3 = S_3 - S_2 = 3 - 4 = -1$

Also, $S_9 = 4 \times 9 - 9^2 = -45$

$S_{10} = 4 \times 10 - 10^2 = -60$ $\therefore$ $a_{10} = S_{10} - S_9 = -60 - (-45) = -15$

Again, $S_n = 4n - n^2$

$\Rightarrow$ $S_{n-1} = 4(n-1) - (n-1)^2 = (4n-4) - (n^2 - 2n + 1) = -n^2 + 6n - 5$

$\therefore$ nth term, $a_n = S_n - S_{n-1} = (4n - n^2) - (-n^2 + 6n - 5)$

$\Rightarrow$ $a_n = 5 - 2n$

$$S_{15} = \frac{15}{2}(2a+14d) = \frac{15}{2}(4+14\times 5) = 555.$$

[NCERT]

Example 14 Find the sum of :

(i) the first 1000 positive integers (ii) the first n positive integers.

X

Solution. (i) Let $S = 1+2+3+\ldots\ldots+1000$

It is an arithmetic series with $a=1$, $d=1$, $n=1000$ and $l=1000$

As $S_n = \frac{n}{2}(a+l)$

$\therefore \quad S_{1000} = \frac{1000}{2}(1+1000) = 500\times 1001 = 500500$

Hence, the sum of the first 1000 positive integers in **500500**.

EX (ii) Let $S_n = 1+2+3+\ldots\ldots+n$

It is an arithmetic series with $a=1$, $d=1$ and $l=n$

S.3 $\therefore \quad S_n = \frac{n}{2}(a+l) = \frac{n}{2}(1+n) = \frac{n(n+1)}{2}$

Hence, the sum of the first n positive integers is given by $S_n = \frac{n(n+1)}{2}$.

[NCE

Q12 **Example** Find the sum of first 40 positive integers divisible by 6.

Solution. The first 40 positive integers divisible by 6 are :

$$6\times 1, 6\times 2, 6\times 3, \ldots, 6\times 40 \quad \text{or} \quad 6, 12, 18, \ldots, 240$$

29/4 It is an AP with $a=6$, $l=240$, $n=40$

$\therefore \quad S_{40} = \frac{n}{2}(a+l)$, (where $n=40$)

X

$$= \frac{40}{2}(6+240) = 20\times 246 = 4920$$

Hence, the sum of the first 40 positive integers divisible by 6 is **4920**.

[N

Q13 **Example** 20 Find the sum of first fifteen multiple of 8.

Solution. The first 15 multiples of 8 are

$$8\times 1, 8\times 2, 8\times 3, \ldots, 8\times 15 \quad \text{or} \quad 8, 16, 24, \ldots, 120$$

It is an AP with $a=8$, $l=120$, $n=15$

$\therefore \quad S_{15} = \frac{n}{2}(a+l)$, (where $n=15$)

$$= \frac{15}{2}(8+120) = 15\times 64 = 960$$

Hence, the sum of first fifteen multiples of 8 is 960.

5.3

Example Find the sum of all the odd numbers between 0 and 50. [N

Solution. The odd numbers between 0 and 50 are : 1, 3, 5,..., 49

It is an AP with $a=1$, $d=3-1=2$, $l=49$

Let $l=a_n=49$

$\Rightarrow \quad a+(n-1)d=49 \quad \Rightarrow \quad 1+(n-1)\times 2=49$

$\Rightarrow \quad n-1=24 \quad \Rightarrow \quad n=25$

$\therefore \quad S_{25}=\frac{n}{2}(a+l)$, (where $n=25$)

$$=\frac{25}{2}(1+49)=25\times 25=625$$

Hence, the sum of all the odd numbers between 0 and 50 is 625.