4. $\cot(x-y) = \dfrac{\cot x \cot y + 1}{\cot y - \cot x}$

T-Ratios of (2v)

1. $\sin 2x = 2\sin x \cos x = \dfrac{2\tan x}{1+\tan^2 x}$.
2. $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
$= \dfrac{1-\tan^2 x}{1+\tan^2 x}$.
3. $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$.

T-Ratios of (3x)

1. $\sin 3x = 3\sin x - 4\sin^3 x$.
2. $\cos 3x = 4\cos^3 x - 3\cos x$.
3. $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1-3\tan^2 x}$.

Sum and Difference of two like ratios :

1. $\sin x + \sin y = 2\sin \dfrac{x+y}{2} \cos \dfrac{x-y}{2}$.
2. $\sin x - \sin y = 2\cos \dfrac{x+y}{2} \sin \dfrac{x-y}{2}$.
3. $\cos x + \cos y = 2\cos \dfrac{x+y}{2} \cos \dfrac{x-y}{2}$.
4. $\cos x - \cos y = -2\sin \dfrac{x+y}{2} \sin \dfrac{x-y}{2}$.

Product of T-Ratios :

1. $2\sin x \cos y = \sin(x+y) + \sin(x-y)$.
2. $2\cos x \sin y = \sin(x+y) - \sin(x-y)$.
3. $2\cos x \cos y = \cos(x+y) + \cos(x-y)$.
4. $2\sin x \sin y = \cos(x-y) - \cos(x+y)$.

(*ii*) The letter S of word 'sugar' indicates sin and cosec of the angle lying in II quadrant are positive and rest of *t*-ratios are negative.

(*iii*) T of word 'to' indicates tan and cot of the angle lying in the third quadrant are positive and rest of the *t*-ratios of such angles are negative.

(*iv*) C of the word coffee shows that cos and sec of the angles lying in IV quadrant are positive and rest of *t*-ratios are negative of such angles.

II. (*i*) For angle $-x$, $\pi - x$, $\pi + x$, $2\pi - x$, $2\pi + x$, the *t*-ratios remains to be the same.

$\sin(-x) = -\sin x$, $\tan(\pi + x) = \tan x$, $\cos(2\pi - x) = \cos x$.

(*ii*) For angles $\frac{\pi}{2} - x$, $\frac{\pi}{2} + x$, $\frac{3\pi}{2} - x$, $\frac{3\pi}{2} + x$ the *t*-ratio changes from cos to sin , tan to cot and sec to cosec and vice versa.

e.g., $\sin\left(\frac{\pi}{2} + x\right) = \cos x$, $\tan\left(\frac{\pi}{2} - x\right) = \cot x$,

$\sec\left(\frac{\pi}{2} + x\right) = -\operatorname{cosec} x$, etc.

T-Ratios of $(x + y)$

1. $\sin(x + y) = \sin x \cos y + \cos x \sin y$
2. $\cos(x + y) = \cos x \cos y - \sin x \sin y$
3. $\tan(x + y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$
4. $\cot(x + y) = \dfrac{\cot x \cot y - 1}{\cot x + \cot y}$

T-Ratios of $(x - y)$

1. $\sin(x - y) = \sin x \cos y - \cos x \sin y$
2. $\cos(x - y) = \cos x \cos y + \sin x \sin y$
3. $\tan(x - y) = \dfrac{\tan x - \tan y}{1 + \tan x \tan y}$

2. Find the degree measures corresponding to the following radian measures. $\left(\text{Use } \pi = \frac{22}{7}\right)$

(*i*) $\frac{11}{16}$ (*ii*) -4 (*iii*) $\frac{5\pi}{3}$ (*iv*) $\frac{7\pi}{6}$

Sol. (*i*) π radians $= \frac{22}{7}$ radians $= 180°$

$\therefore \quad \frac{11}{16}$ radians $= \frac{180}{22} \times 7 \times \frac{11}{16}$ degree

$= \frac{315}{8}$ degrees $= 39\frac{3}{8}$ degrees

$= 39°22'30''$.

Note : $\frac{3°}{8} = \frac{3}{8} \times 60' = \frac{45'}{2} = 22'30''$.

(*ii*) -4 radians $= \left(-4 \times \frac{180}{\pi}\right)$

$= -\left(\frac{4 \times 180 \times 7}{22}\right)$

$= -\left(\frac{2520}{11}\right)$

$= 229°5'27''$ (nearly).

$11 \overline{)\,2520\,(\,229}$

$\underline{22}$

32

$\underline{22}$

100

$\underline{99}$

1

$1 \times 60 = 60$

$11 \overline{)\,60\,(\,5}$ minutes

$\underline{55}$

5 minutes

$5 \times 60 = 300$

$11 \overline{)\,300\,(\,27}$ seconds

$\underline{22}$

80

$\underline{77}$

3

(*iii*) $\frac{5\pi}{3} = \frac{5}{3} \times 180° = 300°$.

(*iv*) $\frac{7\pi}{6} = \frac{7}{6} \times 180° = 210°$.

T-Ratios of $\left(\frac{\pi}{2}+x\right)$

1. $\sin\left(\frac{\pi}{2}+x\right)=\cos x$,
2. $\cos\left(\frac{\pi}{2}+x\right)=-\sin x$,
3. $\tan\left(\frac{\pi}{2}+x\right)=-\cot x$,
4. $\operatorname{cosec}\left(\frac{\pi}{2}+x\right)=\sec x$,
5. $\sec\left(\frac{\pi}{2}+x\right)=-\operatorname{cosec} x$,
6. $\cot\left(\frac{\pi}{2}+x\right)=-\tan x$.

T-Ratios of $(\pi - x)$

1. $\sin(\pi-x)=\sin x$,
2. $\cos(\pi-x)=-\cos x$,
3. $\tan(\pi-x)=-\tan x$,
4. $\operatorname{cosec}(\pi-x)=\operatorname{cosec} x$,
5. $\sec(\pi-x)=-\sec x$,
6. $\cot(\pi-x)=-\cot x$.

T-Ratios of $(\pi + x)$

1. $\sin(\pi+x)=-\sin x$,
2. $\cos(\pi+x)=-\cos x$,
3. $\tan(\pi+x)=\tan x$,
4. $\operatorname{cosec}(\pi+x)=-\operatorname{cosec} x$,
5. $\sec(\pi+x)=-\sec x$,
6. $\cot(\pi+x)=\cot x$.

T-Ratios of $(2\pi - x)$

1. $\sin(2\pi-x)=-\sin x$,
2. $\cos(2\pi-x)=\cos x$,
3. $\tan(2\pi-x)=-\tan x$,
4. $\operatorname{cosec}(2\pi-x)=-\operatorname{cosec} x$,
5. $\sec(2\pi-x)=\sec x$,
6. $\cot(2\pi-x)=-\cot x$.

T-Ratios of $(2\pi + x)$

1. $\sin(2\pi+x)=\sin x$,
2. $\cos(2\pi+x)=\cos x$,
3. $\tan(2\pi+x)=\tan x$,
4. $\operatorname{cosec}(2\pi+x)=\operatorname{cosec} x$,
5. $\sec(2\pi+x)=\sec x$,
6. $\cot(2\pi+x)=\cot x$.

Note : To learn these formulae by heart, we adopt the following method :

I. First of all, we determine the sign of t-ratio under consideration.

The phrase Add Sugar To Coffee is quite helpful. We write Add, Sugar, To, Coffee in I, II, III, IV quadrants respectively.

S (Sugar)	A (Add)
T (To)	C (Coffee)

(*i*) The letter A of word 'add' indicates all the t-ratios of angles lying in the first quadrant are positive.

General solution of an equation :

1. If $\sin x = 0$, then $x = n\pi$, $n \in Z$.
2. If $\cos x = 0$, then $x = (2n + 1)\frac{\pi}{2}$, $n \in Z$.
3. If $\tan x = 0$, then $x = n\pi$, $n \in Z$.
4. $\sin x = \sin y \Rightarrow x = n\pi + (-1)^n y$, $n \in Z$.
5. $\cos x = \cos y \Rightarrow x = 2n\pi \pm y$, $n \in Z$.
6. $\tan x = \tan y \Rightarrow x = n\pi + y$, $n \in Z$,

where y is the principal value of x.

EXERCISE 3.1

1. Find the radian measures corresponding to the following degree measures ?

(*i*) 25° (*ii*) −47°30′ (*iii*) 240° (*iv*) 520°

Sol. We have : $180° = \pi$ radians

$$\therefore \quad 25° = \frac{\pi}{180} \times 25 \text{ radians} = \frac{5\pi}{36} \text{ radians.}$$

(*ii*) $60' = 1°$

$$\therefore \quad 30' = \frac{30°}{60} = \frac{1°}{2}$$

$$\therefore \quad 47°30' = \left(47 + \frac{1}{2}\right) \text{degrees} = \frac{95}{2} \text{ degrees}$$

Now, $180° = \pi$ radians

So, $$-\frac{95°}{2} = \frac{-\pi}{180} \times \frac{95}{2} \text{ radians} = \frac{-19\pi}{72} \text{ radians.}$$

(*iii*) $180° = \pi$ radians.

$$\therefore \quad 240° = \frac{\pi}{180} \times 240 \text{ radians}$$

$$= \frac{4\pi}{3} \text{ radians.}$$

(*iv*) $180° = \pi$ radians

$$\therefore \quad 520° = \frac{\pi}{180} \times 520 \text{ radians} = \frac{26\pi}{9} \text{ radians.}$$