Q 8 $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$ and $B=\begin{bmatrix}-2\\ 3\end{bmatrix}$

Now, $|A|=\begin{vmatrix}2 & -1\\ 3 & 4\end{vmatrix}=8+3=11\neq 0$

$\Rightarrow$ A is non-singular and so given system has a unique solutic

Cofactors of elements of A are given by

$A_{11}=(-1)^{1+1}(4)=4,\ A_{12}=(-1)^{1+2}(3)=-3,$

$A_{21}=(-1)^{2+1}(-1)=1,\ A_{22}=(-1)^{2+2}(2)=2$

$\therefore$ $\text{adj } A=\begin{bmatrix}4 & -3\\ 1 & 2\end{bmatrix}'=\begin{bmatrix}4 & 1\\ -3 & 2\end{bmatrix}$

and $A^{-1}=\dfrac{1}{|A|}(\text{adj } A)=\dfrac{1}{11}\begin{bmatrix}4 & 1\\ -3 & 2\end{bmatrix}$

Solution of given system is given by $X=A^{-1}B$.

$$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{11}\begin{bmatrix}4 & 1\\ -3 & 2\end{bmatrix}\begin{bmatrix}-2\\ 3\end{bmatrix}=\frac{1}{11}\begin{bmatrix}-8+3\\ 6+6\end{bmatrix}=\frac{1}{11}\begin{bmatrix}-5\\ 12\end{bmatrix}=\begin{bmatrix}\frac{-5}{11}\\ \frac{12}{11}\end{bmatrix}$$

$\therefore$ $x=-\dfrac{5}{11},\ y=\dfrac{12}{11}$

9. $4x-3y=3,\ 3x-5y=7.$

Soln.: The given system of equations can be written in the form $AX=B$, where

$A=\begin{bmatrix}4 & -3\\ 3 & -5\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$ and $B=\begin{bmatrix}3\\ 7\end{bmatrix}$

Now, $|A|=\begin{vmatrix}4 & -3\\ 3 & -5\end{vmatrix}=-20+9=-11\neq 0$

$\Rightarrow$ A is a non-singular matrix and so the given system has a unique solution.

Cofactors of elements of A are given by

$A_{11}=(-1)^{1+1}(-5)=-5,\ A_{12}=(-1)^{1+2}(3)=-3,$

$A_{21}=(-1)^{2+1}(-3)=3,\ A_{22}=(-1)^{2+2}(4)=4$

$\therefore$ $\text{adj } A=\begin{bmatrix}-5 & -3\\ 3 & 4\end{bmatrix}'=\begin{bmatrix}-5 & 3\\ -3 & 4\end{bmatrix}$

and $A^{-1}=\dfrac{1}{|A|}(\text{adj } A)=\dfrac{1}{-11}\begin{bmatrix}-5 & 3\\ -3 & 4\end{bmatrix}$

Solution of given system is given by $X=A^{-1}B$.

$$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=-\frac{1}{11}\begin{bmatrix}-5 & 3\\ -3 & 4\end{bmatrix}\begin{bmatrix}3\\ 7\end{bmatrix}$$

$$=-\frac{1}{11}\begin{bmatrix}-15+21\\ -9+28\end{bmatrix}=\frac{1}{-11}\begin{bmatrix}6\\ 19\end{bmatrix}=\begin{bmatrix}\frac{-6}{11}\\ \frac{-19}{11}\end{bmatrix}$$

Hence, $x=\dfrac{-6}{11},\ y=\dfrac{-19}{11}$

10. $5x+2y=3,\ 3x+2y=5.$

Soln.: The given system of equations can be written in the form $AX=B$, where

$A_{22} = (-1)^{2+2}\begin{vmatrix}1 & 1\\ a & 2a\end{vmatrix} = a$, $A_{23} = (-1)^{2+3}\begin{vmatrix}1 & 1\\ a & a\end{vmatrix} = 0$

$A_{31} = (-1)^{3+1}\begin{vmatrix}1 & 1\\ 3 & 2\end{vmatrix} = -1$, $A_{32} = (-1)^{3+2}\begin{vmatrix}1 & 1\\ 2 & 2\end{vmatrix} = 0$,

$A_{33} = (-1)^{3+3}\begin{vmatrix}1 & 1\\ 2 & 3\end{vmatrix} = 1$

$$\therefore \text{ adj } A = \begin{bmatrix}4a & -2a & -a\\ -a & a & 0\\ -1 & 0 & 1\end{bmatrix}' = \begin{bmatrix}4a & -a & -1\\ -2a & a & 0\\ -a & 0 & 1\end{bmatrix}.$$

$$\text{Now, (adj } A)\cdot B = \begin{bmatrix}4a & -a & -1\\ -2a & a & 0\\ -a & 0 & 1\end{bmatrix}\begin{bmatrix}1\\ 2\\ 4\end{bmatrix}$$

$$= \begin{bmatrix}2a-4\\ 0\\ -a+4\end{bmatrix} = \begin{bmatrix}-4\\ 0\\ 4\end{bmatrix} \neq 0 \quad [\because a = 0]$$

Hence, equations are inconsistent with no solution because if $a = 0$, then the third system of equations is not possible.

5. $3x - y - 2z = 2, 2y - z = -1, 3x - 5y = 3.$

Soln.: The system of equations can be written in the form $AX = B$, where

$$A = \begin{bmatrix}3 & -1 & -2\\ 0 & 2 & -1\\ 3 & -5 & 0\end{bmatrix}, X = \begin{bmatrix}x\\ y\\ z\end{bmatrix} \text{ and } B = \begin{bmatrix}2\\ -1\\ 3\end{bmatrix}$$

Now, $|A| = 3(0-5) + 1(0+3) - 2(0-6) = -15 + 3 + 12 = 0$

Here, A is a singular matrix, so we will compute (adj A)B..

For adj A, cofactors of elements of A are given by

$A_{11} = (-1)^{1+1}\begin{vmatrix}2 & -1\\ -5 & 0\end{vmatrix} = -5$, $A_{12} = (-1)^{1+2}\begin{vmatrix}0 & -1\\ 3 & 0\end{vmatrix} = -3$,

$A_{13} = (-1)^{1+3}\begin{vmatrix}0 & 2\\ 3 & -5\end{vmatrix} = -6$,

$A_{21} = (-1)^{2+1}\begin{vmatrix}-1 & -2\\ -5 & 0\end{vmatrix} = -(-10) = 10$,

$A_{22} = (-1)^{2+2}\begin{vmatrix}3 & -2\\ 3 & 0\end{vmatrix} = 6$,

$A_{23} = (-1)^{2+3}\begin{vmatrix}3 & -1\\ 3 & -5\end{vmatrix} = -(-15+3) = 12$,

$A_{31} = (-1)^{3+1}\begin{vmatrix}-1 & -2\\ 2 & -1\end{vmatrix} = 1 + 4 = 5$, $A_{32} = (-1)^{3+2}\begin{vmatrix}3 & -2\\ 0 & -1\end{vmatrix} = 3$,

$A_{33} = (-1)^{3+3}\begin{vmatrix}3 & -1\\ 0 & 2\end{vmatrix} = 6$

$$\therefore \text{ adj } A = \begin{bmatrix}-5 & -3 & -6\\ 10 & 6 & 12\\ 5 & 3 & 6\end{bmatrix}' = \begin{bmatrix}-5 & 10 & 5\\ -3 & 6 & 3\\ -6 & 12 & 6\end{bmatrix}$$

$$\text{Now, (adj } A)\cdot B = \begin{bmatrix}-5 & 10 & 5\\ -3 & 6 & 3\\ -6 & 12 & 6\end{bmatrix}\begin{bmatrix}2\\ -1\\ 3\end{bmatrix}$$

$$= \begin{bmatrix}-10-10+15\\ -6-6+9\\ -12-12+18\end{bmatrix} = \begin{bmatrix}-5\\ -3\\ -6\end{bmatrix} \neq 0$$

Hence, system of equations is inconsistent with no solution.

6. $5x - y + 4z = 5, 2x + 3y + 5z = 2, 5x - 2y + 6z = -1.$

Soln.: The system of equations can be written in the form $AX = B$, where

$$A = \begin{bmatrix}5 & -1 & 4\\ 2 & 3 & 5\\ 5 & -2 & 6\end{bmatrix}, X = \begin{bmatrix}x\\ y\\ z\end{bmatrix} \text{ and } B = \begin{bmatrix}5\\ 2\\ -1\end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix}5 & -1 & 4\\ 2 & 3 & 5\\ 5 & -2 & 6\end{vmatrix} = 5(18+10) + 1(12-25) + 4(-4 \ldots$$

$= 140 - 13 - 76 = 51 \neq 0$

Hence, equations are consistent with a unique solution.

Solve system of linear equations, using matrix method, in questions 7 to 14.

7. $5x + 2y = 4, 7x + 3y = 5.$

Soln.: The given system of equations can be written in the form $AX = B$, where

$$A = \begin{bmatrix}5 & 2\\ 7 & 3\end{bmatrix}, X = \begin{bmatrix}x\\ y\end{bmatrix} \text{ and } B = \begin{bmatrix}4\\ 5\end{bmatrix}$$

Now, $|A| = \begin{vmatrix}5 & 2\\ 7 & 3\end{vmatrix} = 15 - 14 = 1 \neq 0$

$\Rightarrow$ A is non-singular and so given system has a unique solution.

Cofactors of elements of A are given by

$A_{11} = (-1)^{1+1}(3) = 3$, $A_{12} = (-1)^{1+2}(7) = -7$,

$A_{21} = (-1)^{2+1}(2) = -2$, $A_{22} = (-1)^{2+2}(5) = 5$

$$\therefore \text{ adj } A = \begin{bmatrix}3 & -7\\ -2 & 5\end{bmatrix}' = \begin{bmatrix}3 & -2\\ -7 & 5\end{bmatrix}$$

$$\text{and } A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{1}\begin{bmatrix}3 & -2\\ -7 & 5\end{bmatrix} = \begin{bmatrix}3 & -2\\ -7 & 5\end{bmatrix}$$

Solution of given system is given by $X = A^{-1}B$.

$$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}3 & -2\\ -7 & 5\end{bmatrix}\begin{bmatrix}4\\ 5\end{bmatrix} = \begin{bmatrix}12 & -10\\ -28 & +25\end{bmatrix} = \begin{bmatrix}2\\ -3\end{bmatrix}$$

Hence, $x = 2, y = -3$

8. $2x - y = -2, 3x + 4y = 3.$

Soln.: The given system of equations can be written in the form $AX = B$, where

Exercise - 4.6

Examine the consistency of the system of equations in Exercises 1 to 6.

1. $x + 2y = 2, 2x + 3y = 3.$

Soln.: The system of equations can be written in the form $AX = B$, where

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

Now, $|A| = 3 - 4 = -1 \neq 0$

Hence, system of equations is consistent.

2. $2x - y = 5, x + y = 4.$

Soln.: The system of equations can be written in the form $AX = B$, where

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Now, $|A| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 + 1 = 3 \neq 0$

Hence, system of equations is consistent.

3. $x + 3y = 5, 2x + 6y = 8.$

Soln.: The system of equations can be written in the form $AX = B$, where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

Now, $|A| = \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = 6 - 6 = 0$

Hence, A is singular matrix. So, we calculate (adj A)B.

$$\text{adj } A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$$

Now, $(\text{adj } A)B = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ 8 \end{bmatrix} = \begin{bmatrix} 30 - 24 \\ -10 + 8 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Hence, equations are inconsistent with no solution.

4. $x + y + z = 1, 2x + 3y + 2z = 2, ax + ay + 2az = 4.$

Soln.: The system of equations can be written in the form $AX = B$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

Now, $|A| = 1(6a - 2a) - 1(4a - 2a) + 1(2a - 3a) = 4a - 3a = a \neq 0$

Two conditions arise:

I : If $a \neq 0$ then $|A| \neq 0$, hence the system of equations is consistent and has a unique solution.

II : If $a = 0$, then $|A| = 0$. So, we need to calculate adj A.

Cofactors of elements of A are given by

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 2 \\ a & 2a \end{vmatrix} = 4a,\quad A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 2 \\ a & 2a \end{vmatrix} = -2a,$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & 3 \\ a & a \end{vmatrix} = -a,\quad A_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 1 \\ a & 2a \end{vmatrix} = -a,$$