

Q16 **Example 10** A sum of ₹700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹20 less than its preceding prize, find the value of each of the prizes. [NCERT]

Solution. Let ₹ a be the value of first cash prize. As each cash prize is ₹20 less than the preceding one, so the common difference is -20 .

8 Here, $d = -20$, $n = 7$, $S_7 = 700$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

S13 $\therefore S_7 = \frac{7}{2}[2a + 6 \times (-20)] = 700 \quad \Rightarrow 2a - 120 = 200 \quad \Rightarrow 2a = 320$

$\therefore a = 160$

Hence, the values of 7 cash prizes will be ₹160, ₹140, ₹120, ₹100, ₹80, ₹60 and ₹40.

30/4 **Example 11** Yasmeen saves ₹32 during the first month, ₹36 in the second month and ₹40 in third month. If she continues to save in this manner, in how many months will she save ₹2000? [Exemplar Problem]

Solution. Yasmeen's savings in successive months : ₹32, ₹36, ₹40, ... form an AP.

Here, $a = 32$, $d = 36 - 32 = 4$

Let $S_n = \frac{n}{2}[2a + (n-1)d] = 2000$

$\Rightarrow \frac{n}{2}[2 \times 32 + (n-1) \times 4] = 2000 \quad \Rightarrow \quad n[16 + (n-1)] = 1000$

$\Rightarrow n^2 + 15n - 1000 = 0 \quad \Rightarrow \quad n^2 + 40n - 25n - 1000 = 0$

$\Rightarrow (n-25)(n+40) = 0$

$\Rightarrow n = 25 \quad \text{or} \quad n = -40$

As n cannot be negative, so $n = 25$

Hence, Yasmeen will save ₹2000 in 25 months.

Q15 **Example 12** A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows : ₹200 for first day, ₹250 for the second day, ₹300 for the third day, etc. The penalty for each succeeding day being ₹50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days? [NCERT]

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Example 10 A spiral is made up of successive semicircles, with centres alternatively at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 2.0 cm, as shown in Fig. 8.6. What is the total length of such a spiral made up of thirteen consecutive semicircles ? Take $\left(\pi = \frac{22}{7}\right)$

Solution. $R_1 = 0.5$ cm, $R_2 = 1.0$ cm, $R_3 = 1.5$ cm
 These radii form an AP with $a = 0.5$, $d = 1.0 - 0.5 = 0.5$

Length of spiral made up of 13 consecutive semicircles

$$= \pi R_1 + \pi R_2 + \pi R_3 + \dots + \pi R_{13}$$

$$= \pi(R_1 + R_2 + R_3 + \dots + R_{13})$$

$$= \pi(0.5 + 1.0 + 1.5 + \dots + 13\text{th term})$$

$$= \pi \left[\frac{13}{2} [2(0.5) + (13-1)(0.5)] \right] = \pi \left[\frac{13}{2} [1 + 12(0.5)] \right] = \pi \left[\frac{13}{2} \times 7 \right] = \frac{22}{7} \times \frac{13}{2} \times 7 = 143 \text{ cm}$$

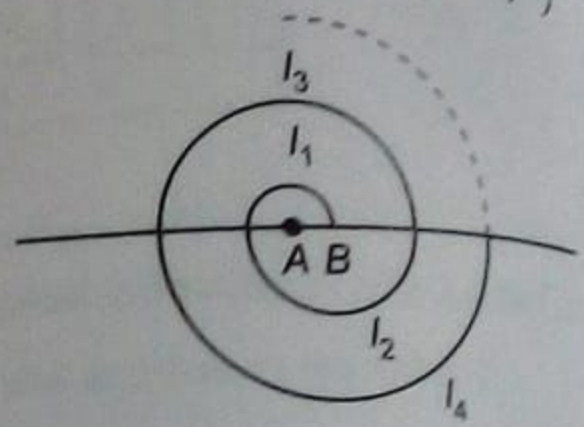


Figure 8.6

X { 5.3 } 30/4

Solution. The penalties (in ₹) for successive days are : ₹200, ₹250, ₹300,...

Here, $a = 200$, $d = 50$, $n = 30$

$$\therefore S_{30} = \frac{30}{2} [2a + (30 - 1)d] = 15 [2 \times 200 + 29 \times 50] = 15 [400 + 1450] = 15 \times 1850 = 27750.$$

Hence, a delay of 30 days will cost ₹27750 as penalty to the contractor.

Example 12 In a school, students thought of planting trees in and around the school to reduce noise pollution and air pollution. It was decided that the number of trees that each section of each class will plant will be the same as the class in which they are studying e.g., a section of I class will plant 1 tree, a section of II class will plant 2 trees and so on and a section of XII class will plant 12 trees. There are three sections of each class. How many trees will be planted by the students ? [NCERT]

Solution. Clearly, the number of trees planted by each section from classes I to XII will be

$$1, 2, 3, 4, \dots, 12$$

Here, $a = 1$, $d = 1$, $n = 12$

$$\therefore S_{12} = \frac{12}{2} [2a + (n - 1)d] = 6 [2 \times 1 + (12 - 1) \times 1] = 6 [2 + 11] = 6 \times 13 = 78$$

As there are 3 sections in each class, so the total number of trees planted by the students

$$= 3 \times 78 = 234.$$

Solution. Let n be the number of rows. Then,
 $20 + 19 + 18 + \dots$ upto n terms = 200

As $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\therefore 200 = \frac{n}{2}[2 \times 20 + (n-1) \times (-1)] \Rightarrow 400 = n[40 - n + 1]$$

$$\Rightarrow n^2 - 41n + 400 = 0$$

$$\Rightarrow (n-25)(n-16) = 0$$

$$\Rightarrow n = 25 \text{ or } n = 16$$

If $n = 25$, then number of logs in the top row,

$$a_{25} = 20 + 24 \times (-1) = -4, \text{ which is not possible.}$$

$$\Rightarrow n = 25 \text{ is rejected}$$

$$\therefore n = 16$$

Number of logs placed in the top (16th) row will be

$$a_{16} = 20 + 15 \times (-1) = 20 - 15 = 5$$

Hence, the logs are placed in 16 rows and the top row has 5 logs.

Example 16 In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Fig. 8.2).

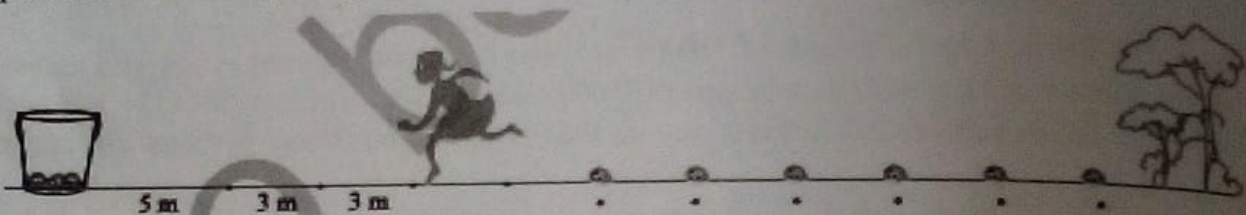


Figure 8.2

A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[NCERT]

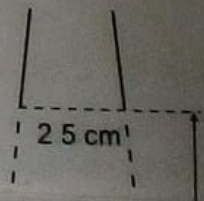
Solution. To pick up the first potato, second potato, third potato, fourth potato, etc., the total distances (in metres) run by the competitor are respectively given by

$$2 \times 5, 2 \times (5+3), 2 \times (5+3+3), 2 \times (5+3+3+3), \dots \text{ or } 10, 16, 22, 28, \dots$$

It is an AP with $a = 10$, $d = 16 - 10 = 6$, $n = 10$

$$\therefore S_{10} = \frac{10}{2}[2 \times 10 + (10-1) \times 6] = 5(20 + 54) = 370$$

Hence, the competitor has to run a total distance of 370 m for dropping all the 10 potatoes in the bucket.



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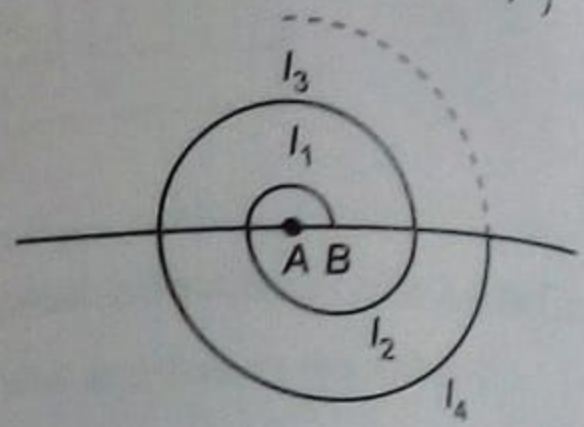


Figure 8.6