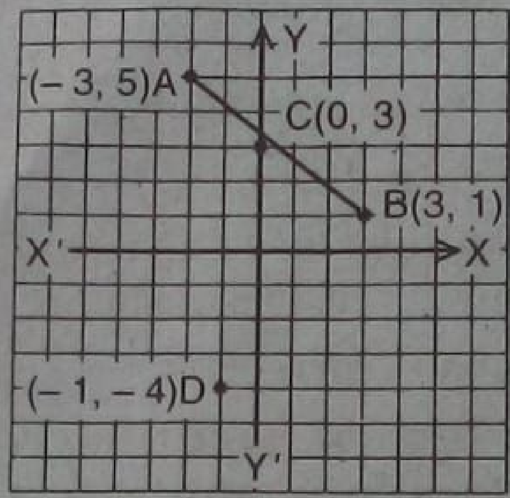
Also, diagonals AC and BD are equal.

$\therefore$ The quadrilateral ABCD is a square.

(*ii*) Let A(– 3, 5), B(3, 1), C(0, 3) and D(– 1, – 4) be the given points. Plot these points as shown.



Clearly, the points A, C and B are collinear. So, no quadrilateral is formed by these points.

(*iii*) Let A(4, 5), B(7, 6), C(4, 3) and D(1, 2) be the given points. Then,

AB

$$AB = \sqrt{(7-4)^2 + (6-5)^2} = \sqrt{9+1} = \sqrt{10}$$

$$BC = \sqrt{(4-7)^2 + (3-6)^2} = \sqrt{9+9} = \sqrt{18}$$

$$CD = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$$

$$DA = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{9+9} = \sqrt{18}$$

$$AC = \sqrt{(4-4)^2 + (3-5)^2} = \sqrt{0+4} = \sqrt{4} = 2$$

and, $BD = \sqrt{(1-7)^2 + (2-6)^2} = \sqrt{36+16} = \sqrt{52}$

Clearly, AB = CD, BC = DA and AC $\neq$ BD.

$\therefore$ The quadrilateral ABCD is a parallelogram.

Q9

$$QP = QR \Rightarrow QP^2 = QR^2$$

$$\Rightarrow (5-0)^2 + (-3-1)^2 = (x-0)^2 + (6-1)^2$$

$$\Rightarrow 25 + 16 = x^2 + 25$$

$$\Rightarrow x^2 = 16 \Rightarrow x = \pm 4$$

Thus, R is (4, 6) or (– 4, 6).

Now, QR = Distance between Q(0, 1) and R(4, 6)

$$= \sqrt{(4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$$

Also, QR = Distance between Q(0, 1) and R(– 4, 6)

$$= \sqrt{(-4-0)^2 + (6-1)^2} = \sqrt{16+25} = \sqrt{41}$$

and, PR = Distance between P(5, – 3) and R(4, 6)

$$= \sqrt{(4-5)^2 + (6+3)^2} = \sqrt{1+81} = \sqrt{82}$$

Also, PR = Distance between P(5, – 3) and R(– 4, 6)

$$= \sqrt{(-4-5)^2 + (6+3)^2} = \sqrt{81+81} = 9\sqrt{2}$$

10. Find a relation between x and y such that the point (x, y) is equidistant from the points (3, 6) and (– 3, 4).

Sol. Let the point $P(x, y)$ be equidistant from the points A(3, 6) and B(– 3, 4)

i.e., $PA = PB$

$$\Rightarrow PA^2 = PB^2$$

$$\Rightarrow (x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$\Rightarrow x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$$

$$\Rightarrow -6x - 6x - 12y + 8y + 36 - 16 = 0$$

$$\Rightarrow -12x - 4y + 20 = 0$$

$3x + y - 5 = 0$

Also, $AC = \sqrt{(9-3)^2 + (4-4)^2}$

$= \sqrt{36+0} = \sqrt{36} = 6$

and, $BD = \sqrt{(6-6)^2 + (1-7)^2}$

$= \sqrt{0+36} = \sqrt{36} = 6$

$\Rightarrow AC = BD = 6$

Thus, the four sides are equal and the diagonals are also equal. Therefore, ABCD is a square.

Hence, Champa is correct.

6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer :

(i) (– 1, – 2), (1, 0), (– 1, 2), (– 3, 0)

(ii) (– 3, 5), (3, 1), (0, 3), (– 1, – 4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Sol. (i) Let A(– 1, – 2), B(1, 0), C(– 1, 2) and D(– 3, 0) be the given points. Then,

$$AB = \sqrt{(1+1)^2 + (0+2)^2} = \sqrt{4+4} = \sqrt{8}$$

$$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = \sqrt{8}$$

$$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = \sqrt{8}$$

$$DA = \sqrt{(-1+3)^2 + (-2-0)^2} = \sqrt{4+4} = \sqrt{8}$$

$$AC = \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+16} = 4$$

and, $BD = \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{16+0} = 4$

Clearly, four sides AB, BC, CD and DA are equal.

7. Find the point on the x-axis which is equidistant from (2, – 5) and (– 2, 9).

Sol. Since the point on x-axis have its ordinate = 0, so $P(x, 0)$ is any point on the x-axis.

Since $P(x, 0)$ is equidistant from $A(2, -5)$ and $B(-2, 9)$

$\therefore$

$$PA = PB \Rightarrow PA^2 = PB^2$$

$$\Rightarrow \quad (x-2)^2 + (0+5)^2 = (x+2)^2 + (0-9)^2$$

$$\Rightarrow \quad x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$$

$$\Rightarrow \quad -4x - 4x = 81 - 25$$

$$\Rightarrow \quad -8x = 56$$

$$\Rightarrow \quad x = \frac{56}{-8} = -7$$

$\therefore$ The point equidistant from given points on the x-axis is (– 7, 0).

8. Find the values of y for which the distance between the points P(2, – 3) and Q(10, y) is 10 units.

Sol. $P(2, -3)$ and $Q(10, y)$ are given points such that PQ = 10 units.

But,
$$PQ = \sqrt{(10-2)^2 + (y+3)^2}$$

$$\Rightarrow \quad 10 = \sqrt{64 + y^2 + 6y + 9}$$

$$\Rightarrow \quad 100 = 73 + y^2 + 6y$$

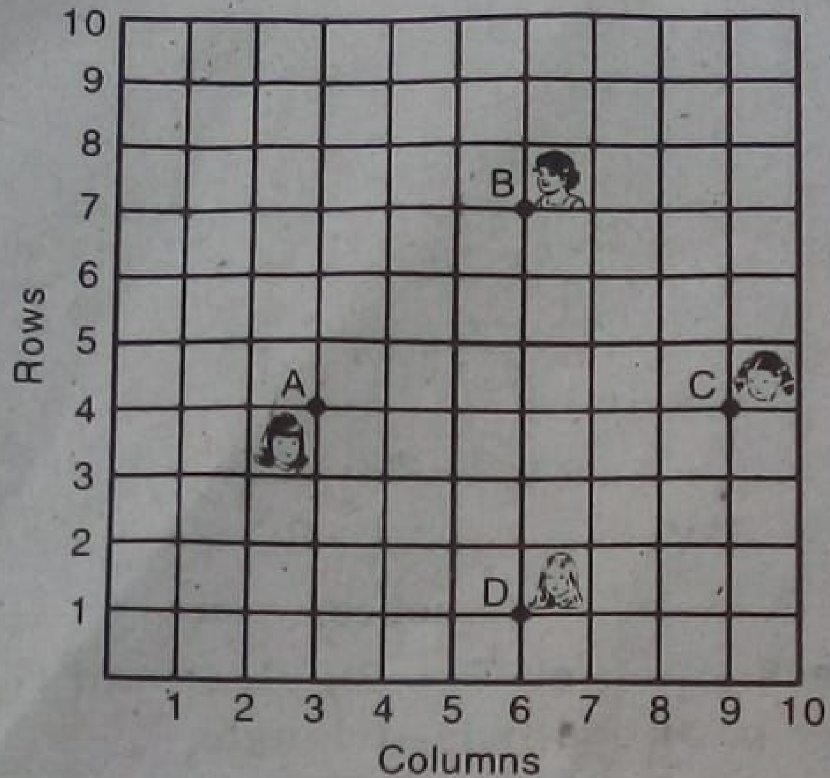
$$\Rightarrow \quad y^2 + 6y - 27 = 0 \quad \Rightarrow \quad (y+9)(y-3) = 0$$

$$\Rightarrow \quad y = -9 \text{ or } 3$$

Thus, the possible values of y are – 9 or 3.

9. If Q(0, 1) is equidistant from P(5, – 3) and R(x, 6), find the values of x. Also find the distances QR and PR.

Sol. Since the point Q(0, 1) is equidistant from P(5, – 3) and R(x, 6) therefore,



Sol. Clearly from the figures, the coordinates of points A, B, C and D are (3, 4), (6, 7), (9, 4) and (6, 1).

By using distance formula, we get

$$AB = \sqrt{(6-3)^2 + (7-4)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(9-6)^2 + (4-7)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CD = \sqrt{(6-9)^2 + (1-4)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

and, $$DA = \sqrt{(3-6)^2 + (4-1)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\Rightarrow \quad AB = BC = CD = DA = 3\sqrt{2}$$