EXERCISE 3.3

XI

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Prove that :

1. $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$

2. $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

3. $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$

4. $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$

Solutions to questions 1 – 4 :

1. L.H.S. $=\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$

$$=\left(\sin\frac{\pi}{6}\right)^2+\left(\cos\frac{\pi}{3}\right)^2-\left(\tan\frac{\pi}{4}\right)^2$$

$$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-1^2$$

$$\left[\because \sin\frac{\pi}{6}=\frac{1}{2}, \cos\frac{\pi}{3}=\frac{1}{2}, \tan\frac{\pi}{4}=1\right]$$

$$=\frac{1}{4}+\frac{1}{4}-1=\frac{1}{2}-1$$

$$=\frac{-1}{2}=\text{R.H.S.}$$

5. Find the value of :

(i) $\sin 75°$ **(ii)** $\tan 15°$

Sol. (i) $\sin 75° = \sin(45° + 30°)$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$

$[\sin(A+B) = \sin A \cos B + \cos A \sin B]$

$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}.$$

(ii) $\tan 15° = \tan(45° - 30°)$

$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$$

$$\left[\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}\right]$$

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \times \frac{1}{\sqrt{3}}} \qquad \left[\begin{array}{l} \because \tan 45° = 1 \\ \tan 30° = \frac{1}{\sqrt{3}} \end{array}\right]$$

$$= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2}{3-1} = \frac{3+1-2\sqrt{3}}{2}$$

$$= \frac{4-2\sqrt{3}}{2} = 2 - \sqrt{3}.$$

Prove the following ...

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2. L.H.S. $= 2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$$= 2\left(\sin\frac{\pi}{6}\right)^2 + \left(\operatorname{cosec}\frac{7\pi}{6}\right)^2\left(\cos\frac{\pi}{3}\right)^2$$

$$= 2\left(\frac{1}{2}\right)^2 + \left[\operatorname{cosec}\left(\pi + \frac{\pi}{6}\right)\right]^2\left(\frac{1}{2}\right)^2$$

$$= 2\times\frac{1}{4} + \left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$$

[$\because$ *cosec* $(\pi + \theta) = -$ *cosec* θ]

$$= \frac{1}{2} + (2)^2\frac{1}{4} = \frac{1}{2} + 1 = \frac{3}{2} = \text{R.H.S.}$$

3. L.H.S. $= \cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$

$$= \left(\cot\frac{\pi}{6}\right)^2 + \operatorname{cosec}\left(\pi - \frac{\pi}{6}\right) + 3\left(\tan\frac{\pi}{6}\right)^2$$

$$= (\sqrt{3})^2 + \operatorname{cosec}\frac{\pi}{6} + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + 2 + 3\times\frac{1}{3} = 3 + 2 + 1 = 6 = \text{R.H.S.}$$

4. L.H.S. $= 2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$

$$= 2\left(\sin\frac{3\pi}{4}\right)^2 + 2\left(\cos\frac{\pi}{4}\right)^2 + 2\left(\sec\frac{\pi}{3}\right)^2$$

$$= 2\left(\sin\left[\pi - \frac{\pi}{4}\right]\right)^2 + 2\times\left(\frac{1}{\sqrt{2}}\right)^2 + 2\times(2^2)$$

$$= 2\left(\sin\frac{\pi}{4}\right)^2 + 2\times\frac{1}{2} + 8$$

$$= 2\times\left(\frac{1}{\sqrt{2}}\right)^2 + 1 + 8 = 2\times\frac{1}{2} + 1 + 8$$

$$= 1 + 1 + 8 = 10 = \text{R.H.S.}$$