Explanation of Photoelectric Effect using Quantum Theory

According to Einstein, when a photon strikes a metal surface, some of its energy is used up to eject the electron from the metal atom (equal to the energy binding the electron with the nucleus) and the remaining energy is given to eject electron in the form of kinetic energy. This may be expressed as :

Energy of striking photon = Binding energy + Kinetic energy of ejected electron

This means that a certain minimum amount of energy corresponding to the binding energy is necessary to detach the electron from the metal. Thus, when a photon of energy $h\nu$, strikes a metal surface, (Fig. 21), some of its energy, called threshold energy x is used up to remove the electron from the surface and the remaining energy is imparted to the ejected electron as kinetic energy $\frac{1}{2}mv^2$. Therefore,

$$h\nu = x + \frac{1}{2}mv^2$$

If the threshold frequency is ν_0, then the threshold energy, $x = h\nu_0$ so that

$$h\nu = h\nu_0 + \frac{1}{2}mv^2$$

or $$\frac{1}{2}mv^2 = h\nu - h\nu_0$$

Therefore, the kinetic energy of ejected electron,

$$\text{K.E.}\left(\frac{1}{2}mv^2\right) = h(\nu - \nu_0)$$

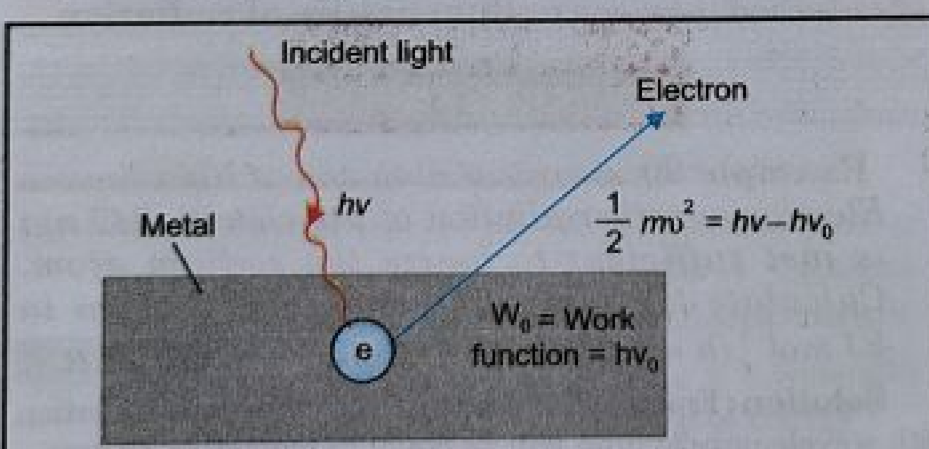

Fig. 21. Einstein explanation for photoelectric effect.

The threshold energy, x ($h\nu_0$) is also called **work function, w_0**. It is characteristic of a particular metal.

The values of work function (w_0) for some common metals are given below :

Metal	Li	Na	K	Cs	Mg	Cu	Ag
w_0 (eV)	2.42	2.3	2.25	2.14	3.7	4.8	4.3

The above equation can easily explain the experimental facts as discussed below :

(*i*) If the frequency of radiation ν, is less than ν_0, no electrons will be ejected whatever the intensity of radiation may be. *The threshold frequency is different for different metals.*

(*ii*) If frequency of the striking radiation is more than the threshold value (*i.e.*, $\nu > \nu_0$), the excess energy $(h\nu - h\nu_0)$ is imparted to the ejected electron as kinetic energy. Thus, as the frequency of radiation increases, the kinetic energy of the electron increases.

(*iii*) Each photon can eject the electron. On increasing the intensity of light of a given frequency, the number of photons striking the surface is increased but the kinetic energy remains unchanged. Consequently, the greater intensity of light of given frequency (more than ν_0) results into more electrons being ejected but their kinetic energy does not change.

REMEMBER

The energy acquired by an electron when it is accelerated through a potential difference of 1 volt is called one electron volt (1eV)

$$1eV = \text{charge on electron} \times 1 \text{ volt}$$
$$= 1.602 \times 10^{-19} CV = 1.602 \times 10^{-19} J$$

Dual nature of Electromagnetic radiation. *Light has been regarded as waves to explain the phenomena of reflection, refraction, diffraction, etc. However, in order to explain the photoelectric effect, Einstein regarded the light as tiny particles called photons. In other words, light behaves like waves as well as like particles. Since light is a kind of radiation, it may be concluded that all radiations behave like waves as well as like particles. Such a wave like and particle like nature of radiation is known as* **dual nature of radiation.**

SOLVED EXAMPLES

Example 20.

Electromagnetic radiation of wavelength 242 nm is just sufficient to ionize the sodium atom. Calculate the ionization energy of sodium in kJ mol^{-1} ($h = 6.626 \times 10^{-34}$ J s). **N.C.E.R.T.**

Solution : Energy possessed by electromagnetic radiation with wavelength 242 nm will be equal to ionization energy,

$$E = h\nu = \frac{hc}{\lambda}$$
$$h = 6.626 \times 10^{-34} \text{ J s}$$
$$c = 3.0 \times 10^8 \text{ m}, \lambda = 242 \times 10^{-9} \text{ m}$$
$$\therefore \quad E = \frac{(6.626 \times 10^{-34} \text{ J s}) \times (3.0 \times 10^8 \text{ m s}^{-1})}{(242 \times 10^{-9} \text{ m})}$$
$$= 8.21 \times 10^{-19} \text{ J}$$

Ionization energy per mol $= 8.21 \times 10^{-19} \text{ J} \times 6.023 \times 10^{23}$
$= 494 \times 10^3 \text{ J} = \mathbf{494 \text{ kJ mol}^{-1}}$.

Example 21.

Calculate the minimum amount of energy that the photons must possess to eject electrons from cesium metal. The threshold frequency of cesium metal is 4.6×10^{14} s^{-1} ($h = 6.63 \times 10^{-34}$ J s).

Solution : [Threshold frequency (ν_0) is the minimum frequency that the photons must possess to eject electrons from metals. Therefore, the energy corresponding to ν_0 is the minimum energy required, also called work function].

Threshold frequency, $\nu_0 = 4.6 \times 10^{14}$ s^{-1}

Minimum energy required to eject the electrons from cesium metal,

$$E_0 = h\nu_0$$
$$h = 6.63 \times 10^{-34} \text{ J s}$$
$$\therefore \quad E_0 = (6.63 \times 10^{-34} \text{ J s}) \times (4.6 \times 10^{14} \text{ s}^{-1})$$
$$= 6.63 \times 4.6 \times 10^{-20} \text{ J} = \mathbf{3.05 \times 10^{-19} \text{ J}}$$

Example 22.

Calculate the kinetic energy of the ejected electron when ultra-violet radiation of frequency 1.6×10^{15} s^{-1} strikes the surface of potassium metal. Threshold frequency of potassium is 5×10^{14} s^{-1} ($h = 6.63 \times 10^{-34}$ J s).

Solution : K.E. of the ejected electron is given by

$$\text{K.E.} = h\nu - h\nu_0 = h(\nu - \nu_0)$$
$$\nu = 1.6 \times 10^{15} \text{ s}^{-1}, \nu_0 = 5 \times 10^{14} \text{ s}^{-1}$$
$$\therefore \quad \text{K.E.} = (6.63 \times 10^{-34} \text{ J s}) \times (1.6 \times 10^{15} - 5 \times 10^{14}) \text{ s}^{-1}$$
$$= (6.63 \times 10^{-34} \text{ J s}) \times (11 \times 10^{14} \text{ s}^{-1})$$
$$= \mathbf{7.29 \times 10^{-19} \text{ J}}$$

Example 23.

A 25 watt bulb emits monochromatic yellow light of wavelength 0.57 μm. Calculate the rate of emission of quanta per second. **N.C.E.R.T.**

Solution : Energy of one photon,

$$E = h\nu = \frac{hc}{\lambda}$$
$$h = 6.626 \times 10^{-34} \text{ J s}$$
$$c = 3.0 \times 10^8 \text{ m}, \lambda = 0.57 \text{ μm}$$
$$= 0.57 \times 10^{-6} \text{ m}$$
$$E = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^8}{0.57 \times 10^{-6}}$$
$$= 3.487 \times 10^{-19} \text{ J}$$

Now, 25 Watt = 25 J/s

$$\therefore \text{ No. of photons emitted per second} = \frac{25}{3.487 \times 10^{-19}}$$
$$= \mathbf{7.17 \times 10^{19} \text{ s}^{-1}}.$$

Example 24.

When light of wavelength 470 nm falls on the surface of potassium metal, electrons are emitted with a velocity of 6.4×10^4 m s^{-1}. What is the minimum energy required per mole to remove an electron from potassium metal ?

Solution : Velocity of emitted electrons

$$= 6.4 \times 10^4 \text{ m s}^{-1}$$

Kinetic energy of emitted electrons K.E. $= \frac{1}{2}mv^2$

$$= \frac{1}{2} \times 9.1 \times 10^{-31} \times (6.4 \times 10^4)^2$$
$$\therefore \quad = 1.864 \times 10^{-21} \text{ kg m}^2 \text{ s}^{-2}$$
$$= 1.864 \times 10^{-21} \text{ J}$$

Energy of photon,

$$E = h\nu = \frac{hc}{\lambda}$$

$$E = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{470 \times 10^{-9}}$$

$= 4.23 \times 10^{-19}$ J

Minimum energy required to remove an electron = $h\nu_0$

Now, K.E. = $h\nu - h\nu_0$

or $h\nu_0 = h\nu$ – K.E.

$= 4.23 \times 10^{-19} - 1.864 \times 10^{-21}$

$= 421.14 \times 10^{-21}$ J

Minimum energy required per mole

$= 421.14 \times 10^{-21} \times 6.023 \times 10^{23}$

$= 253.6 \times 10^{3}$ J mol^{-1}

or $=$ **253.6 kJ mol^{-1}**

❑ Example 25.

When electromagnetic radiation of wavelength 300 nm strikes a metal surface of sodium, electrons are emitted with a kinetic energy of 1.68 $\times 10^5$ J mol^{-1}. What is the minimum energy needed to remove an electron from sodium ? What is the maximum wavelength that will cause a photoelectron to be emitted ? **N.C.E.R.T.**

Solution : Energy of the striking photon,

$$E = h\nu = \frac{hc}{\lambda}$$

$h = 6.626 \times 10^{-34}$ J s, $c = 3.0 \times 10^{8}$ m s^{-1},

$\lambda = 300$ nm $= 300 \times 10^{-9}$ m

$$\therefore \quad E = \frac{6.626 \times 10^{-34}\,\text{J s} \times 3.0 \times 10^{8}\,\text{m s}^{-1}}{300 \times 10^{-9}\,\text{m}}$$

$= 6.626 \times 10^{-19}$ J

Kinetic energy of emitted electrons = 1.68×10^{5} J mol^{-1}

$$\text{Kinetic energy of emitted one electron} = \frac{1.68 \times 10^{5}}{6.022 \times 10^{23}}$$

$= 2.79 \times 10^{-19}$ J

Now,

Energy of striking photon = Minimum energy required to eject electron + Kinetic energy of electron or Minimum energy required for ejection of an electron

$= 6.626 \times 10^{-19}$ J $- 2.79 \times 10^{-19}$ J

$=$ **3.84×10^{-19} J**

The wavelength which will cause photoelectron emission,

$$\lambda = \frac{hc}{E}$$

$$= \frac{6.626 \times 10^{-34}\,\text{J s} \times 3.0 \times 10^{8}\,\text{m s}^{-1}}{3.84 \times 10^{-19}\,\text{J}}$$

$= 5.17 \times 10^{-7}$ m

or $= 517 \times 10^{-9}$ m $=$ **517 nm**.

❑ Example 26.

A photon of wavelength 4.0×10^{-7}m strikes on metal surface, the work function of the metal being 2.13 eV. Calculate :

(i) the energy of photon (eV)

(ii) the kinetic energy of the emission and

(iii) the velocity of the photoelectron emitted

(1eV = 1.6020×10^{-19} J) **N.C.E.R.T.**

Solution: (*i*) Energy of photon

$$E = h\nu = \frac{hc}{\lambda}$$

$\lambda = 4.0 \times 10^{-7}$ m, $\quad c = 3.0 \times 10^{8}$ m s^{-1}

$h = 6.626 \times 10^{-34}$ J s,

$$\therefore \quad E = \frac{(6.626 \times 10^{-34}\,Js) \times (3.0 \times 10^{8}\,m)}{(4.0 \times 10^{-7}\,m)}$$

$= 4.97 \times 10^{-19}$ J

$$\text{or} \quad = \frac{4.97 \times 10^{-19}}{1.6020 \times 10^{-19}} = \mathbf{3.10\ eV.}$$

(*ii*) Kinetic energy of emission

K.E. = $h\nu - h\nu_0$

$h\nu = 3.10$ eV and $h\nu_0$ (work function) = 2.13eV

$\therefore$ K.E. = 3.10 – 2.13 = 0.97 eV

(*iii*) Now, K.E. $= \frac{1}{2} mv^2$

or $\frac{1}{2} mv^2 = 0.97$ eV $= 0.97 \times 1.602 \times 10^{-19}$ J

or $\frac{1}{2} \times (9.11 \times 10^{-31}\text{kg})\, v^2 = 0.97 \times 1.602 \times 10^{-19}$ J

$$v^2 = \frac{2 \times 0.97 \times 1.602 \times 10^{-19}\,\text{kg m}^2\text{s}^{-2}}{9.11 \times 10^{-31}\,\text{kg}}$$

$= 34.12 \times 10^{10}$ m^2 s^{-2}

$\therefore \quad v =$ **5.84×10^{5} m s^{-1}**

Practice Problems

- **18.** Find energy of each of photons which
  - (*i*) have wavelength of 0.50 Å
  - (*ii*) correspond to light of frequency 3×10^{15} Hz.
- **19.** Calculate the energy of one of the photons of a beam of light having wavelength 25.0 μm.
- **20.** In the ultra-violet region of atomic spectrum of hydrogen, a line is obtained at 1050 Å. Calculate the energy of one million photons of this wavelength.
- **21.** How many photons of light having a wavelength 400 nm are necessary to provide 1 J of energy ?
- **22.** A quantum of light having energy E has wavelength equal to 7200 Å. Calculate the frequency of light which corresponds to energy equal to one quarter of E.
- **23.** A photochemical reaction requires 9.6×10^{-16} J energy per molecule. Calculate the number of photons per molecule of light with wavelength 250 nm that is just sufficient to initiate the reaction.
- **24.** The threshold frequency for a metal 'X' is 7.0×10^{14} s^{-1}. Calculate the kinetic energy of an electron emitted when radiation of frequency 1.0×10^{15} s^{-1} strikes the metal. **N.C.E.R.T.**

25. When a photon of frequency $1.0 \times 10^{15} s^{-1}$ was allowed to hit a metal surface, an electron having 1.988×10^{-19} J of kinetic energy was emitted. Calculate the threshold frequency of the metal. Show that an electron will not be emitted if a photon of wavelength 600 nm hits the metal surface.

(*NCERT Exemplar Problem*)

26. Electrons are emitted with zero velocity from a metal surface when it is exposed to radiation of wavelength 6800Å. Calculate threshold frequency (v_0) and work function (w_0) of the metal.

27. The longest wavelength doublet absorption transition is observed at 589 and 589.6 nm. Calculate the frequency of each transition and energy difference between two excited states.

N.C.E.R.T.

Answers to Practice Problems

18. (*i*) 3.98×10^{-15} J (*ii*) 1.98×10^{-18} J
19. 7.96×10^{-21} J
20. 1.89×10^{-12} J
21. 2.01×10^{18} photons
22. 1.04×10^{14} sec^{-1}
23. 121 photons.
24. 1.99×10^{-19} J
25. $v_0 = 6.988 \times 10^{14} s^{-1}$. No, because the frequency of the photon of striking radiation is less than v_0
26. $4.41 \times 10^{14} s^{-1}$, 2.92×10^{-19} J
27. 3.44×10^{-22} J

Hints & Solutions on page 216

ATOMIC SPECTRA

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