Q3 Example To conduct Sports Day activities, in your rectangular shaped school ground $ABCD$, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in Fig. 9.20. Niharika runs $\frac{1}{4}$th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$ th the distance AD on the eighth line and posts a red flag. What is the distance between both the flags ? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag ? [NCERT]

X

EX 7.2

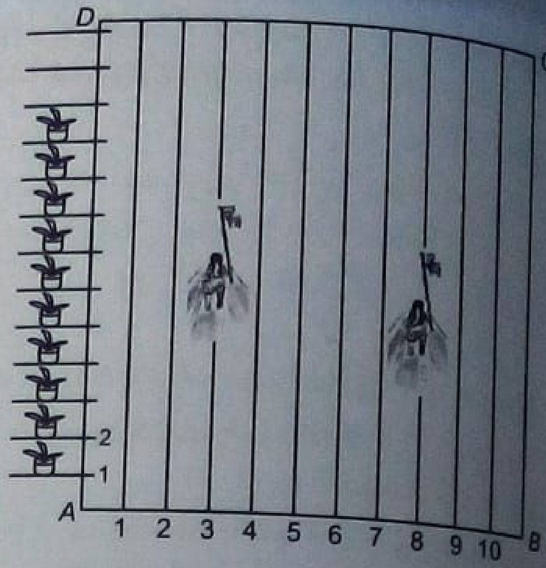


FIGURE 9.20

Solution. Let AB be x-axis and AD be y-axis with A as the origin.

Niharika runs along 2nd line and covers a distance

$$= \frac{1}{4} \times AD = \frac{1}{4} \times 100 = 25 \text{ m, and posts a green flag there}$$

$\therefore$ Coordinates of green flag are $(2, 25)$.

Preet runs along 8th line and covers a distance

$$= \frac{1}{5} \times AD = \frac{1}{5} \times 100 = 20 \text{ m, and posts a red flag there.}$$

$\therefore$ Coordinates of red flag are $(8, 20)$.

Distance between green and real flags

$$= \sqrt{(8-2)^2 + (20-25)^2}$$

$$= \sqrt{36+25} = \sqrt{61} \text{ m}$$

Coordinates of midpoint of the line segment joining green and real flags are

$$\left(\frac{2+8}{2}, \frac{25+20}{2}\right) = \left(5, \frac{45}{2}\right)$$

Hence, Rashmi should post the blue flag on the 5th line at a distance of 22.5 m from x-axis.

EX-7.2 X 56-

Solved Examples

Type I : Problems based on Finding the Point of Division and Division Ratio

Q1 **Example 1** Find the co-ordinates of the point which divides the join of $(-1,7)$ and $(4,-3)$ in the ratio 2:3. [NCERT]

Solution. Let the point $P(x,y)$ divide the joint of points $A(-1,7)$ and $B(4,-3)$ in the ratio $2:3$.

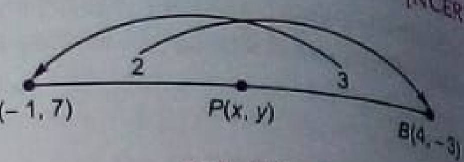


FIGURE 9.9

Here, $x_1=-1,\ y_1=7,\ x_2=4,\ y_2=-7,$

$m_1=2,\ m_2=3$

Using section formula, we get

$$x=\frac{m_1x_2+m_2x_1}{m_1+m_2}=\frac{2\times4+3\times(-1)}{2+3}=\frac{5}{5}=1$$

$$y=\frac{m_1y_2+m_2y_1}{m_1+m_2}=\frac{2\times(-3)+3\times7}{2+3}=\frac{15}{5}=3$$

Hence, the required point is **(1, 3)**.

Q2 **Example 2** Find the coordinates of the points of trisection of the line joining the points $(4,-1)$ and $(-2,-3)$. [NCERT]

Solution. Let P and Q be the points of trisection of line segment AB joining the points $A(4,-1)$ and $B(-2,-3)$. Then, $AP=PQ=QB$.

FIGURE 9.10

Clearly, P divides AB internally in the ratio 1 : 2. So, the coordinates of P are

$$\left(\frac{1\times(-2)+2\times4}{1+2},\frac{1\times(-3)+2(-1)}{1+2}\right)\text{ or }\left(2,-\frac{5}{3}\right)$$

Also, Q divides AB internally in the ratio $2:1$. So, the coordinates of Q are

$$\left(\frac{2\times(-2)+1\times4}{2+1},\frac{2\times(-3)+1\times(-1)}{2+1}\right)\text{ or }\left(0,-\frac{7}{3}\right).$$

Hence, the coordinates of the points of trisection of AB are $\left(2,-\frac{5}{3}\right)$ and $\left(0,-\frac{7}{3}\right)$.

Q4 **Example 3** Find the ratio in which the line segment joining the points $(-3,10)$ and $(6,-8)$ is divided by $(-1,6)$. [NCERT]

Solution. Let the point $P(-1,6)$ divide the line segment joining points $A(-3,10)$ and $B(6,-8)$ in the ratio $k:1$. Using section formula, the coordinates of P are given by

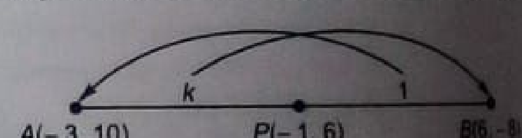


FIGURE 9.11

$$x=\frac{kx_2+x_1}{k+1} \quad\text{and}\quad y=\frac{ky_2+y_1}{k+1}$$

$$-1=\frac{6k-3}{k+1} \qquad 6=\frac{-8k+10}{k+1}$$

$$-k-1=6k-3 \qquad 6k+6=-8k+10$$

$$-7k=-2 \qquad 14k=4$$

$$k=\frac{2}{7} \qquad k=\frac{2}{7}$$

Hence, the required ratio is $\frac{2}{7}:1$ or $2:7$.

NOTE As there is only one unknown, only one equation is sufficient to find the value of k.

Section Formula 7·2 Unit-7

The coordinates of the point $P(x, y)$ which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are $\left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

Proof. Let XOX' and YOY' be the coordinate axes. Let AB be the line segment joining the end points $A(x_1, y_1)$ and $B(x_2, y_2)$. Let the point $P(x, y)$ divide AB internally in the ratio $m_1 : m_2$. Then,

$$\frac{AP}{PB} = \frac{m_1}{m_2}$$

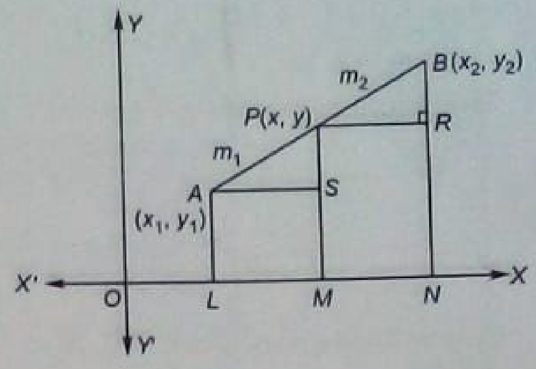


FIGURE 9.8

Draw $AL \perp OX$, $PM \perp OX$, $BN \perp OX$, $AS \perp PM$ and $PR \perp BN$. Then,

$$OL = x_1,\quad ON = x_2,\quad OM = x$$

and $$AL = y_1,\quad BN = y_2,\quad PM = y$$

$\therefore$
$$AS = LM = OM - OL = x - x_1$$
$$PR = MN = ON - OM = x_2 - x$$
$$BR = BN - RN = BN - PM = y_2 - y$$
$$PS = PM - SM = PM - AL = y - y_1$$

Now, $\Delta ASP \sim \Delta PRB$ [AA similarity]

$\therefore$
$$\frac{AS}{PR} = \frac{AP}{PB}$$
$$\frac{x - x_1}{x_2 - x} = \frac{m_1}{m_2}$$
$$m_2x - m_2x_1 = m_1x_2 - m_1x$$
$$(m_1 + m_2)x = m_1x_2 + m_2x_1$$
$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2}$$

and

$$\frac{PS}{BR} = \frac{AP}{PB}$$
$$\frac{y - y_1}{y_2 - y} = \frac{m_1}{m_2}$$
$$m_2y - m_2y_1 = m_1y_2 - m_1y$$
$$(m_1 + m_2)y = m_1y_2 + m_2y_1$$
$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2}$$

Hence, the coordinates of P are $\left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$.

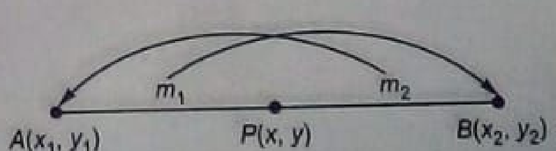
This formula is known as **section formula**.

NOTE

1. **Midpoint formula.** The midpoint of a line segment divides the line segment in the ratio 1 : 1. Hence, the coordinates of the midpoint P of line segment joining points $A(x_1, y_1)$ and $B(x_2, y_2)$ are

$$\left(\frac{1x_1 + 1x_2}{1+1}, \frac{1y_1 + 1y_2}{1+1}\right) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

2. **For remembering section formula** for internal division, the figure given alongside is helpful. Clearly, m_1 is multiplied by the coordinate *away from it* and similarly m_2 is multiplied by the coordinate away from it, and then the sum is divided by $m_1 + m_2$.



3. **The general point of division.** If we are required to find the ratio in which the point P divides the line segment AB, it is convenient to take the ratio $k : 1$ instead of $m_1 : m_2$. Then the coordinates of the point of division will be

$$\left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right)$$