$\frac{t}{5}$ 15

Solutions to questions 10 – 15 :

10. L.H.S. $= \sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x$
$= \cos(n+2)x \cos(n+1)x + \sin(n+2)x \sin(n+1)x$

Put $(n+2)x = A$, $(n+1)x = B$.

L.H.S. $= \cos A \cos B + \sin A \sin B$
$= \cos(A - B)$
$= \cos[(n+2)x - (n+1)x]$
$= \cos x =$ R.H.S.

11. L.H.S. $= \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$

Put $\frac{3\pi}{4} + x = \theta$, $\frac{3\pi}{4} - x = \phi$.

$\therefore$ L.H.S. $= \cos\theta - \cos\phi$

$$= -2\sin\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}$$

$$= -2\sin\left(\frac{\frac{3\pi}{4} + x + \frac{3\pi}{4} - x}{2}\right)\sin\left(\frac{\left(\frac{3\pi}{4} + x\right) - \left(\frac{3\pi}{4} - x\right)}{2}\right)$$

$$= -2\sin\left(\frac{\frac{3\pi}{2}}{2}\right)\sin\left(\frac{2x}{2}\right)$$

$$= -2\sin\frac{3\pi}{4}\sin x$$

$$= -2\sin\left(\pi - \frac{\pi}{4}\right)\sin x = -2\sin\frac{\pi}{4}\sin x$$

$$= -2 \times \frac{1}{\sqrt{2}}\sin x = -\sqrt{2}\sin x = \text{R.H.S.}$$

12. L.H.S. $= \sin^2 6x - \sin^2 4x$

We apply the formula $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$.

[Proof : $\sin(A+B)\sin(A-B)$
$= (\sin A \cos B + \cos A \sin B)(\sin A \cos B - \cos A \sin B)$
$= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B$
$= \sin^2 A(1 - \sin^2 B) - (1 - \sin^2 A)\sin^2 B$

$= \sin^2 A - \sin^2 B = \sin(6x+4x)\sin(6x-4x)$
$= \sin 10x \sin 2x$

8. L.H.S. $= \dfrac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\dfrac{\pi}{2} + x\right)}$

Ex- 3.3

Now, $\cos(\pi + x) = -\cos x$, $\cos(-x) = \cos x$,

$\sin(\pi - x) = \sin x$, $\cos\left(\dfrac{\pi}{2} + x\right) = -\sin x$.

$$\therefore \text{ L.H.S.} = \frac{-\cos x \times \cos x}{\sin x \times (-\sin x)} = \frac{-\cos^2 x}{-\sin^2 x} = \frac{\cos^2 x}{\sin^2 x}$$

$$= \cot^2 x = \text{R.H.S.}$$

9. L.H.S. $= \cos\left(\dfrac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\dfrac{3\pi}{2} - x\right) + \cot(2\pi + x)\right]$

Now, $\cos\left(\dfrac{3\pi}{2} + x\right) = \sin x$, $\cos(2\pi + x) = \cos x$,

$\cot\left(\dfrac{3\pi}{2} - x\right) = \tan x$, $\cot(2\pi + x) = \cot x$.

$$\therefore \text{ L.H.S.} = \sin x \cos x\,[(\tan x) + \cot x]$$

$$= \sin x \cos x\left[\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right]$$

$$= \sin x \cos x\left[\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right]$$

$$= (\sin x \cos x)\frac{1}{\sin x \cos x} \qquad [\because \sin^2 x + \cos^2 x = 1]$$

$$= 1 = \text{R.H.S.}$$

Prove the following :

10. $\sin(n + 1)x \sin(n + 2)x + \cos(n + 1)x \cos(n + 2)x = \cos x$

11. $\cos\left(\dfrac{3\pi}{4} + x\right) - \cos\left(\dfrac{3\pi}{4} - x\right) = -\sqrt{2}\sin x$

12. $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$

13. $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

14. $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

15. $\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$

8. $\dfrac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\dfrac{\pi}{2} + x\right)} = \cot^2 x$

EX-3.3

9. $\cos\left(\dfrac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\dfrac{3\pi}{2} - x\right) + \cot(2\pi + x)\right] = 1$

Solutions to questions 6 – 9 :

6. L.H.S. $= \cos\left(\dfrac{\pi}{4} - x\right)\cos\left(\dfrac{\pi}{4} - y\right) - \sin\left(\dfrac{\pi}{4} - x\right)\sin\left(\dfrac{\pi}{4} - y\right)$

Put $\dfrac{\pi}{4} - x = A$ and $\dfrac{\pi}{4} - y = B$, we get :

$$\begin{aligned}
\text{L.H.S.} &= \cos A \cos B - \sin A \sin B \\
&= \cos(A + B) \\
&= \cos\left[\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right] = \cos\left[\frac{\pi}{2} - (x + y)\right]
\end{aligned}$$

$$\left[\because \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta\right]$$

$$= \sin(x + y) = \text{R.H.S.}$$

7. L.H.S. $= \dfrac{\tan\left(\dfrac{\pi}{4} + x\right)}{\tan\left(\dfrac{\pi}{4} - x\right)} = \dfrac{\dfrac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x}}{\dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}}$

$$= \frac{\dfrac{1 + \tan x}{1 - \tan x}}{\dfrac{1 - \tan x}{1 + \tan x}} = \frac{1 + \tan x}{1 - \tan x} \times \frac{1 + \tan x}{1 - \tan x} \quad \left[\begin{aligned} &\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}, \\ &\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \\ &\text{and } \tan\frac{\pi}{4} = 1. \end{aligned}\right]$$

$$= \frac{(1 + \tan x)^2}{(1 - \tan x)^2} = \text{R.H.S.}$$